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INVESTIGATION OF FREEZING SOILS USING TIME DOMAIN REFLECTOMETRY

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Abstract

The report describes the use of the time-domain reflectometry technique to determine soil freezing characteristic curves. In addition the influence of partial saturation, bulk density variation and percent clay on those characteristic curves is examined. Preliminary evaluation suggests that excess ice contents may also be inferred by comparing the apparent dielectric constant for the icy soil with that measured at normal field density.

Résumé

Ce rapport décrit l'utilisation de la technique de réflectométrie par intervalles de temps pour déterminer les courbes caractéristiques de congélation de sols. En plus l'influence de la non-saturation, des variations de la densité globale et du pourcentage d'argile sur ces courbes caractéristiques out été examinés. Les résultats préliminaires suggèrent qu'une teneur en glace excessive peut être déduite en comparant la constante diélectrique apparente d'un sol contenant de la glace avec la constante mesurée sur le sol à sa densité normale sur le terrain.

FINAL REPORT

INVESTIGATION OF FREEZING SOILS USING TIME DOMAIN REFLECTOMETRY

for the

Department of Energy, Mines and Resources

Earth Physics Branch

by

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Introduction

An empirical relationship between the apparent dielectric constant, K_a, and the volumetric liquid water content of a mineral soil, determined by Topp et al. (1980), was found previously to apply to freezing soils (Smith and Patterson, 1980). Experiments have been continued to obtain further data on the use of the TDR technique to determine soil freezing characteristic curves (θ_{uf} vs. T ^{o}C). Data have been obtained for a variety of soils, and compared to values determined by other methods where possible. These results lend further support to the applicability to frozen soils of the empirical relationship determined for unfrozen soils by Topp et al. (1980).

Data from the TDR experiments have also been used to evaluate the influence of the percent saturation, the variation in bulk density, and the percent clay content on the θ_{uf} vs. T ^{o}C relationship.

Preliminary instrumentation of a laboratory soil column has been carried out to examine whether the TDR technique can be used to monitor changes in soil hydrology during freezing experiments. For closed system freezing, which involves only the internal redistribution of water in the soil profile, the TDR technique was used to measure the water loss in the unfrozen zone. This was then compared to the water gain (by ice accumulation) in the frozen zone, determined gravimetrically at the end of the experiment.

Clearly, it would be more desirable to be able to directly measure the total water content (ice plus water) in the frozen zone, during such experiments. Further, in open system freezing experiments

there may not be any desiccation of the unfrozen zone (due to a constant water supply), and one clearly needs a measure of the total water content in the frozen zone. To accomplish this, the TDR technique could be combined with a gammaray apparatus, for example. However, a single system, if possible, would seem preferable.

No simple extension to the TDR technique has yet been found to permit direct measurement of either ice content or ice plus water content. Other methods do exist for determining the electrical properties of ice and water (e.g., loss via tangent, capacitance, or resistivity measurement); however, incorporating them with the TDR technique may not be possible. Investigation of this is continuing.

In the meantime, a method of inferring maximum possible ice content, from TDR measurement of K_a, has been devised. Since K_a is highly sensitive to the volumetric liquid water content, then the value will decrease as ice replaces water in a unit volume of soil. Therefore, the effect of excess ice accumulation in a soil will be to lower the value of K_a, measured at some temperature, compared to that for the same soil at normal field density. This difference in K_a can be used to calculate a "maximum possible ice content".

SECTION 2

EXPERIMENTAL PROCEDURES

2.1 Freezing Characteristic Curves

When parallel transmission lines are used, a slurried sample is placed in a PVC tube 20 cm long and 5 cm I.D., and allowed to consolidate over a few days. A balanced parallel line probe of 0.3 cm (1/8 in.) stainless steel rod is then inserted in the sample; a line length of 17.5 cm was used, with a line spacing of 2.5 cm (Figure 1). The PVC tube is covered with a latex rubber membrane and immersed in a circulating ethylene glycol bath, which permits temperature control to $\pm 0.01^{\circ}C$ for extended periods of time. Freezing is initiated by subjecting the sample to a temperature of about $-4^{\circ}C$ for 30 to 45 minutes after it has stabilised near $0^{\circ}C$ for 24 hours. The sample is then ramped through a temperature cycle and K_a (hence θ_{uf}) determined at each temperature. This type of freezing cell can be used for both freezing and thawing cycles, since tube rupture due to freezing is not a problem.

To compile a freezing characteristic curve takes about 2-3 days for a silt loam, allowing 4 or 5 hours for each point to reach equilibrium. One can check for equilibrium by monitoring K_a a number of times at each temperature until a steady TDR trace is obtained. For a heavy clay at temperatures between 0° and $-1^{\circ}C$, it can take up to 24 hours to reach equilibrium in the sample. A complete curve down to $-5^{\circ}C$ can be obtained in 4-6 days, depending on the number of points desired.

2.2 Freezing Column Experiments

One procedure used to obtain data on the rates and amount of water

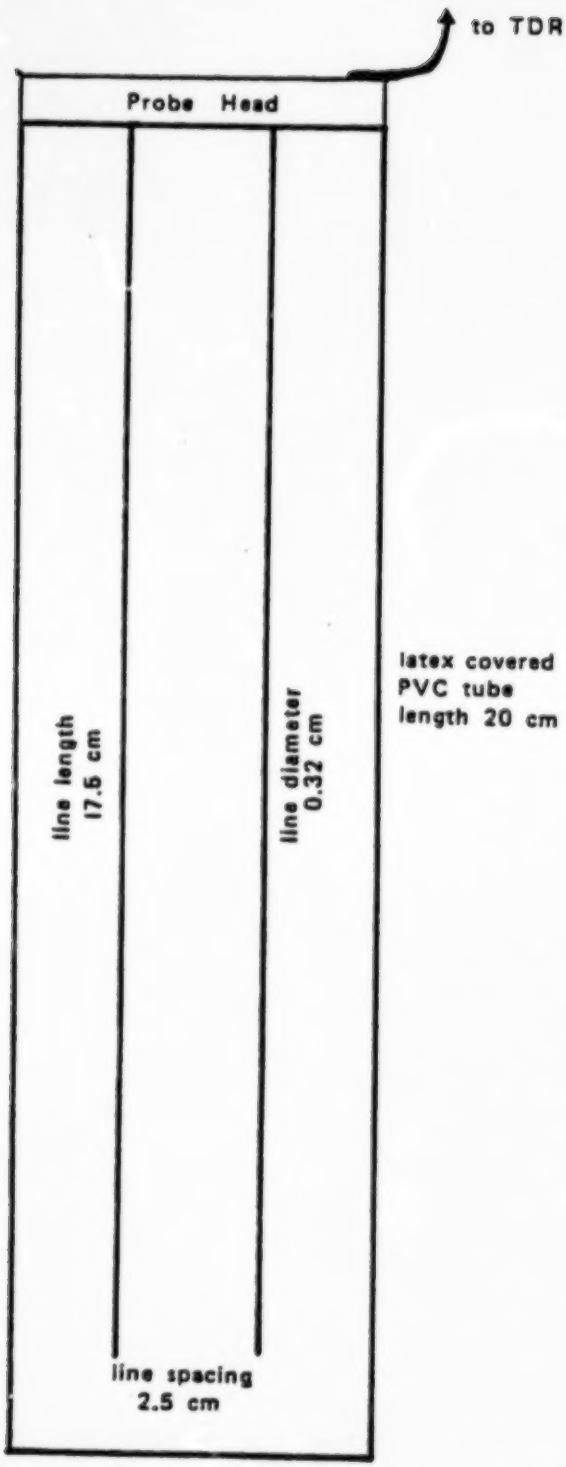


Figure 1 Parallel Transmission Line Freezing Cell

movement toward the freezing front during closed system soil freezing is to replicate soil profiles, freeze them for various time periods and then section the profiles to obtain water content with depth gravimetrically (e.g., Torrance and Williams, 1976; Mageau and Morgenstern, 1980). This replication process is time-consuming and can introduce experimental errors which can make the interpretation of results difficult. Also, this approach cannot be used on undisturbed soil cores obtained from the field. Since the TDR technique is a non-destructive method, the need for duplicating soil profiles is removed and the possibility of instrumenting undisturbed soil cores exists (if the core diameter is large enough, say about 12.5 - 15 cm).

A freezing column, shown in Figure 2, was constructed to permit controlled sample freezing and water content measurement via the TDR technique. Two cooling plates were used to control the end temperatures, with the top plate at sub-freezing temperatures. Balanced parallel transmission lines, installed horizontally at 2.5 cm intervals along the column length, were used for water content measurement. Closed-system freezing experiments have been carried out initially, to examine the use of the TDR technique for monitoring the desiccation of the unfrozen zone as water migrates towards the freezing front.

Soil samples (of Ellwood clay loam and Cavan loam) were slurried, placed in the column and allowed to consolidate for several days. The column was then insulated, placed in a controlled temperature chamber (at ~ 1°C) and the temperature of both cooling plates set to 1°C. Once the sample temperature had stabilized at 1°C, the temperature of the top plate was stepped down to about -4°C to initiate sample freezing. The

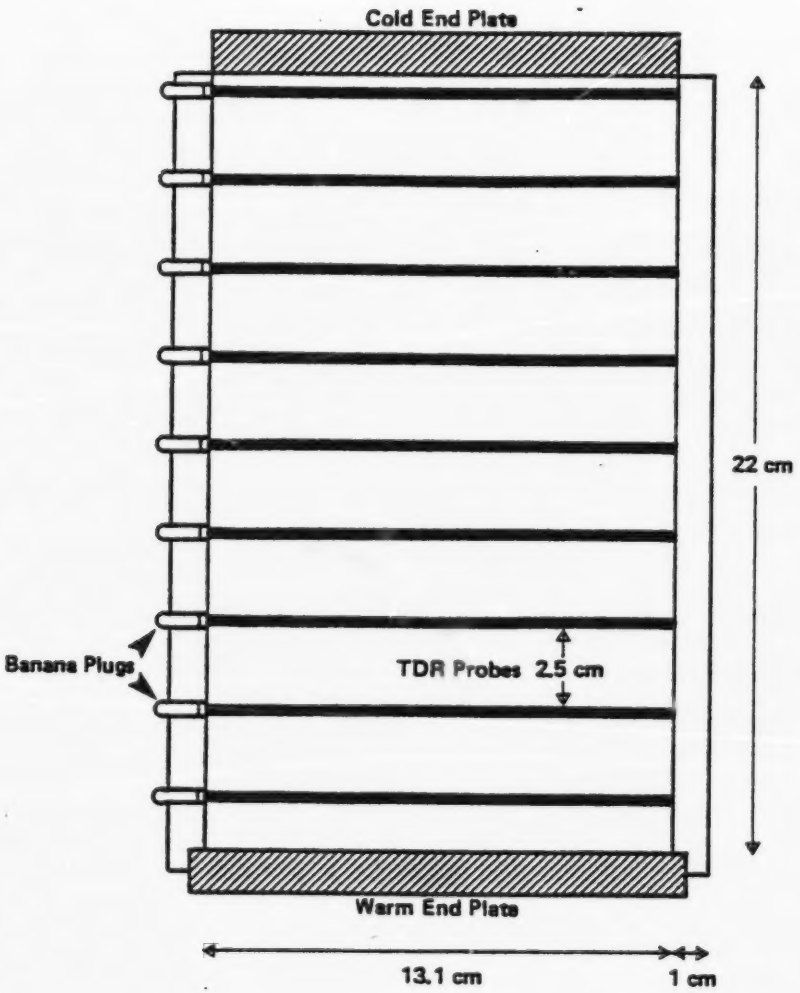


Figure 2 Design for Freezing Column Experiments

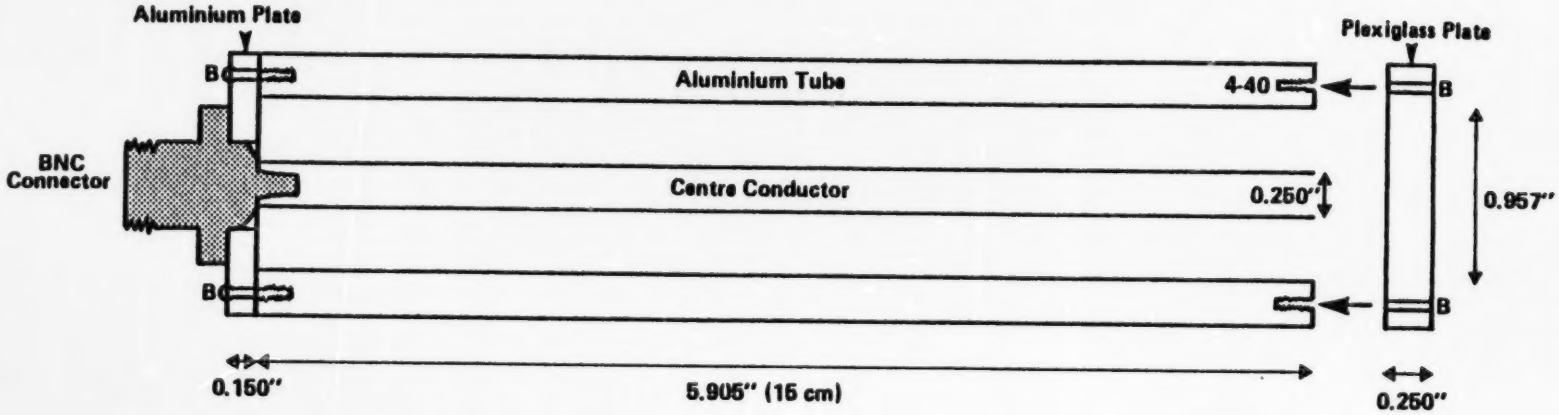
temperature of the top plate was then set to a temperature of -1°C. K_a (and hence θ_v) was measured throughout the unfrozen soil at the beginning and end of each experiment (which lasted about 120 hours). Also, at the end of the experiment the whole column was cored, using a brass ring corer (2.5 cm in length, 5.1 cm I.D.), to determine the water content profile gravimetrically. Some problems were encountered with this where the soil tended to crumble.

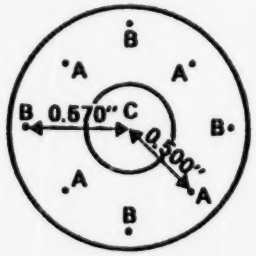
Test results are presented in Section 3.2.

2.3 K_a Determination for Low Density Samples

Experiments were performed to obtain K_a vs T°C data for samples with dry bulk densities less than 1 g cm^{-3}, for example, as in the case of soils containing excess ice. A small coaxial freezing cell was designed for these experiments; it is 15 cm long, has an inside diameter of 2.5 cm and a 0.64 cm ($\frac{1}{4}$ in.) diameter centre conductor (Figure 3). With coaxial freezing cells, care must be taken that the centre conductor is not heaved, thereby possibly breaking the connection at the BNC connector (see Figure 3). To avoid this, a soil slurry is cooled to about 0°C and added in small amounts to the cell. After each addition of soil, the cell is placed in a freezer at -18°C to rapidly freeze the sample. This is repeated until the coaxial cell is filled. The cell is then covered with a latex membrane and placed in the temperature bath.

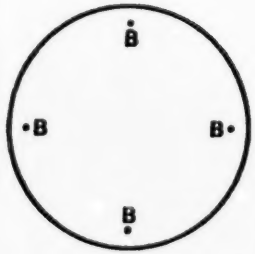
A sample containing excess ice, but without banded ice, can be obtained if a fine-grained super-saturated slurry is used and is added in small amounts to the cell, since freezing will occur before the particles settle out of suspension. Total water content and bulk density





Aluminium Top Plate
0.150" Thick
1.322" O.D.

A 43 Drill Tapped for 4-40 Screws
B 3/32 " Clearance Hole
C 7/16 " Centre Hole



Plexiglass Bottom Plate

Figure 3 Coaxial Line Freezing Cell

were known since all masses and volumes were determined prior to and following each experiment. By comparing K_a values for low bulk density samples to K_a values for samples of "normal" bulk density, some inference about ice content can be made. Experimental results are presented in Section 3.3.

In a number of other experiments, discrete ice lenses were formed by adding water to the cell between soil layers. The ice lenses form impedance contrasts along the transmission line, and produce a signature on the TDR trace (see Section 3.3).

SECTION 3

RESULTS AND DISCUSSION

3.1 Freezing Characteristic Data

Freezing characteristic data were determined for a variety of soils using the TDR technique. θ_{uf} was determined from K_a measurements using an empirical relationship obtained by Topp et al. (1980) for unfrozen mineral soils (Appendix I), which has been found to apply also to freezing soils (Smith and Patterson, 1980).

Two broad groups of soils were selected, sandy and clayey, with several different soils within each group. The results have been examined with respect to grain size characteristics, dry bulk density, and degree of saturation. Grain size characteristics for all samples tested are presented in Table 1. Freezing characteristic data are presented in Figures 4 and 5 for the sandy and clayey soils respectively. The points shown represent the mean value if more than one sample was used. A complete tabulation of all test results is given in Appendix II.

Upon examining Figure 4, there is a great deal of variability in θ_{uf} at temperatures above $-0.5^{\circ}C$ for the sandy soils (a range of about 20% at $-0.1^{\circ}C$). However, the θ_{uf} data at temperatures colder than $-0.5^{\circ}C$ generally agree to within $\pm 2\%$. Previous experiments on the reproducibility of freezing characteristic data for samples of the same soil type showed agreement to well within $\pm 2\frac{1}{2}\%$ in θ_{uf} at all temperatures (Smith and Patterson, 1980). Therefore, it is concluded that the scatter portrayed in Figure 4 at temperatures above $-0.5^{\circ}C$ are due to variations in properties between soil types (e.g., pore size distribution).

TABLE 1

GRAIN SIZE CHARACTERISTCS

	% SAND	% SILT	% CLAY	SOIL DESIGNATION*
Castor	66.1	30.4	3.5	sandy loam
Castor <74u	55.4	40.6	4.0	sandy loam
Garden	64.6	25.1	10.3	sandy loam
Agri	56.9	24.7	17.6	sandy loam
Field	84.8	7.1	8.1	loamy sand
Loeb	35.0	31.7	33.3	clay loam
Allendale	17.7	33.9	48.5	clay
Agri	12.0	45.7	41.7	silty clay
LE2	2.9	58.3	38.8	silty clay loam
LE7	10.8	29.7	59.5	clay
Ellwood	39.8	24.5	35.7	clay loam

* U.S. Department of Agriculture Classification System

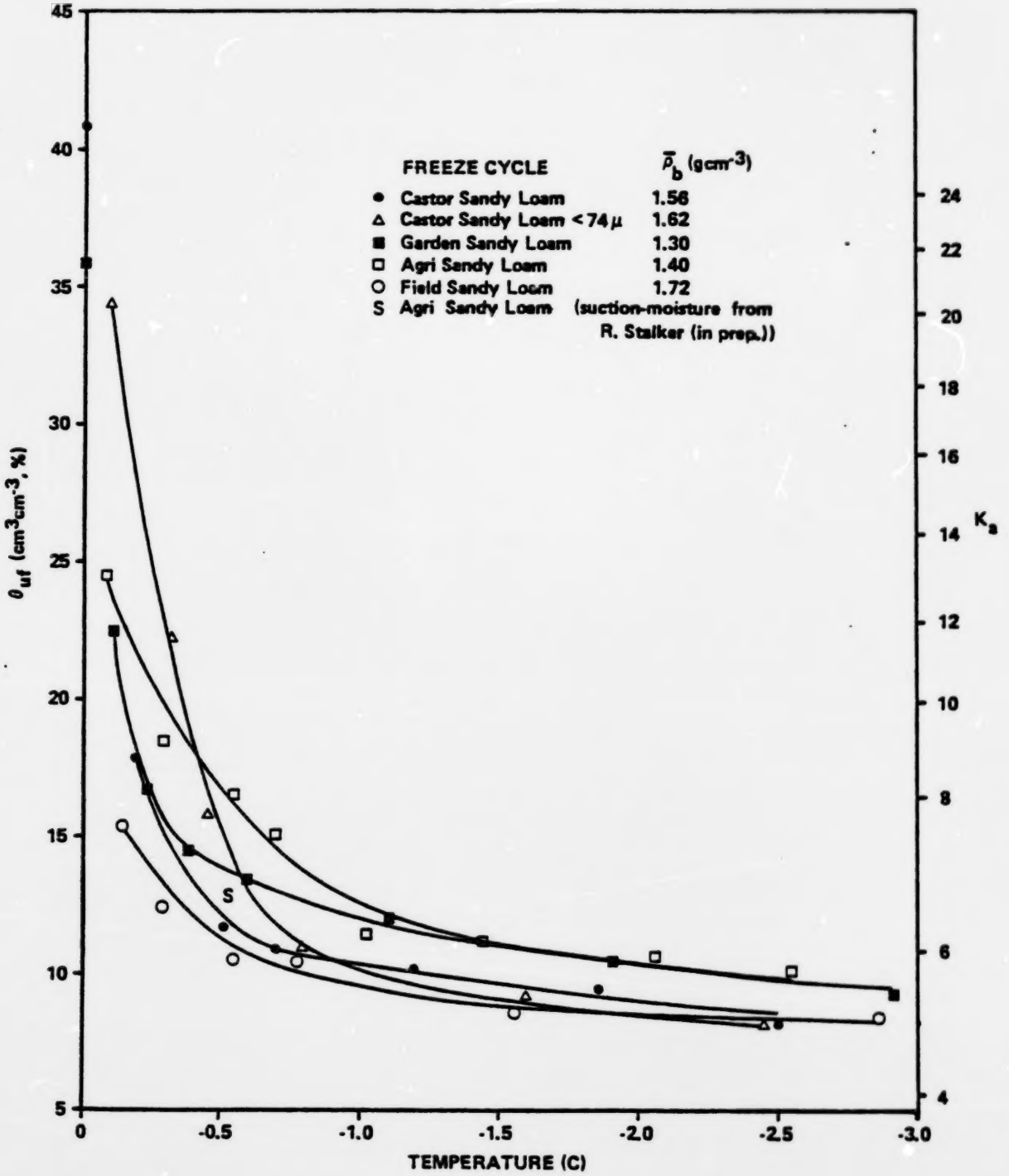


Figure 4 Freezing Characteristic Data: Sandy Soils

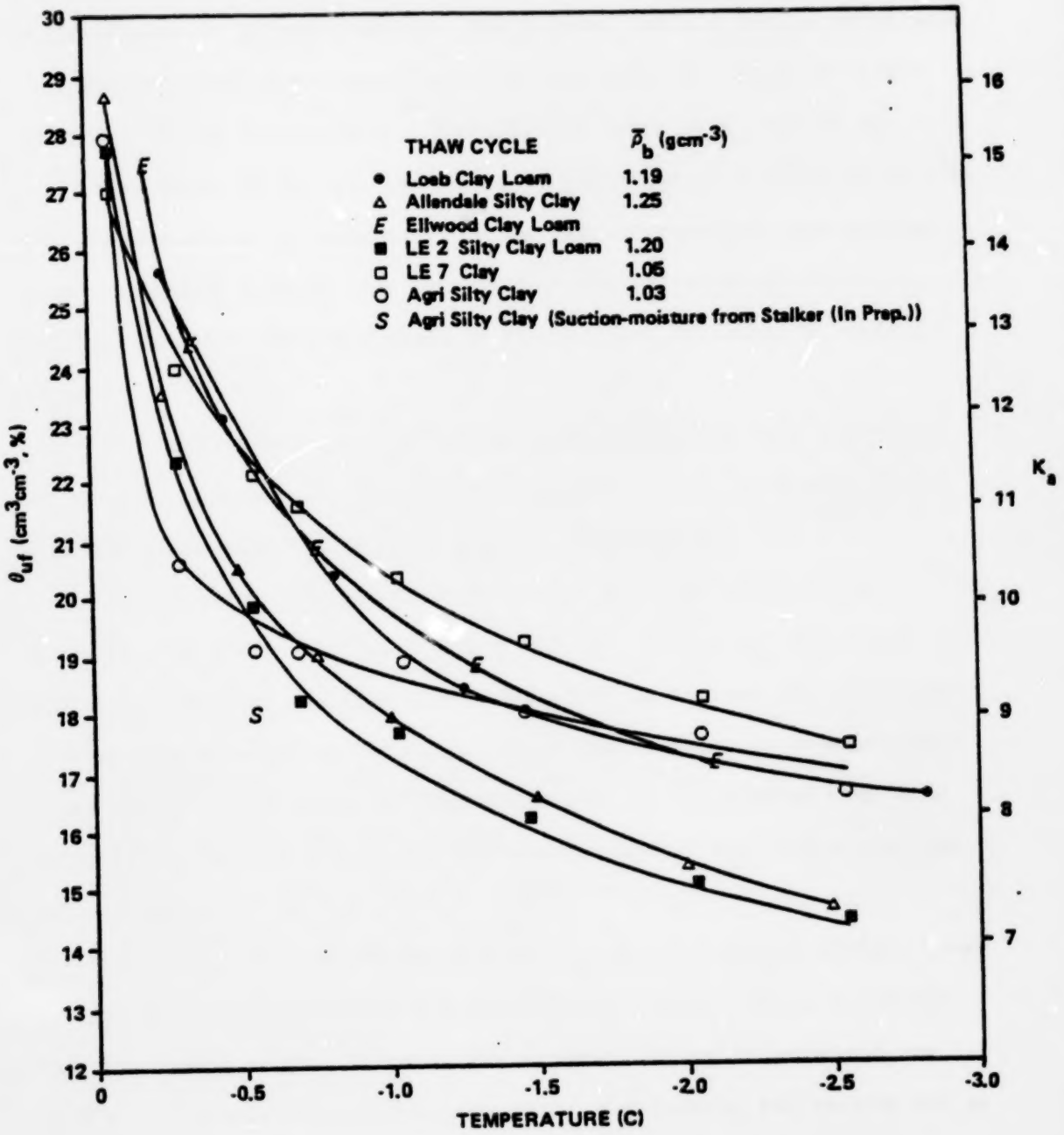


Figure 5 Freezing Characteristic Data: Clayey Soils

The agreement in freezing characteristic curves for various clayey soils (Figure 5) is satisfactory. The Ellwood and Loeb soils, which have very similar grain size characteristics (see Table 1), agree to within 1% in θ_{uf} at all temperatures. Overall, the range in θ_{uf} values at $-0.25^{\circ}C$ is about 5% for all the soils; it decreases to about 3% at $-2.5^{\circ}C$. TDR freezing characteristic data for LE2 and LE7 Norwegian clay samples were compared to similar data derived from suction-moisture tests by Williams (1975). These are shown in Figure 6 and agreement is within 2% in θ_{uf}.

Suction-moisture data for the Agri sandy loam and Agri silty clay at $-0.54^{\circ}C$, obtained from Stalker (in progress), also agree favourably with TDR determined θ_{uf} (Figures 4 and 5, Appendix II).

The freezing characteristic data were examined on the basis of grain size to see if any relationship exists. Dillon and Andersland (1966) and Anderson and Tice (1972) attempted to predict unfrozen water content from specific surface area, which can be measured or estimated; since the clay size fraction contributes most to the surface area of a soil, it was decided to compare TDR measurements of θ_{uf} to the clay content of the soil.

The data for finer textured materials are for warming cycles; those for coarser textured samples are for freezing cycles. Because freeze-thaw hysteresis occurs in soils, the warmest temperature examined was $-0.5^{\circ}C$ (below this, the differences in θ_{uf} for warming and cooling cycles would be small in the coarser textured soils). θ_{uf} data previously determined by the TDR technique, found in Smith and Patterson (1980), were also used.

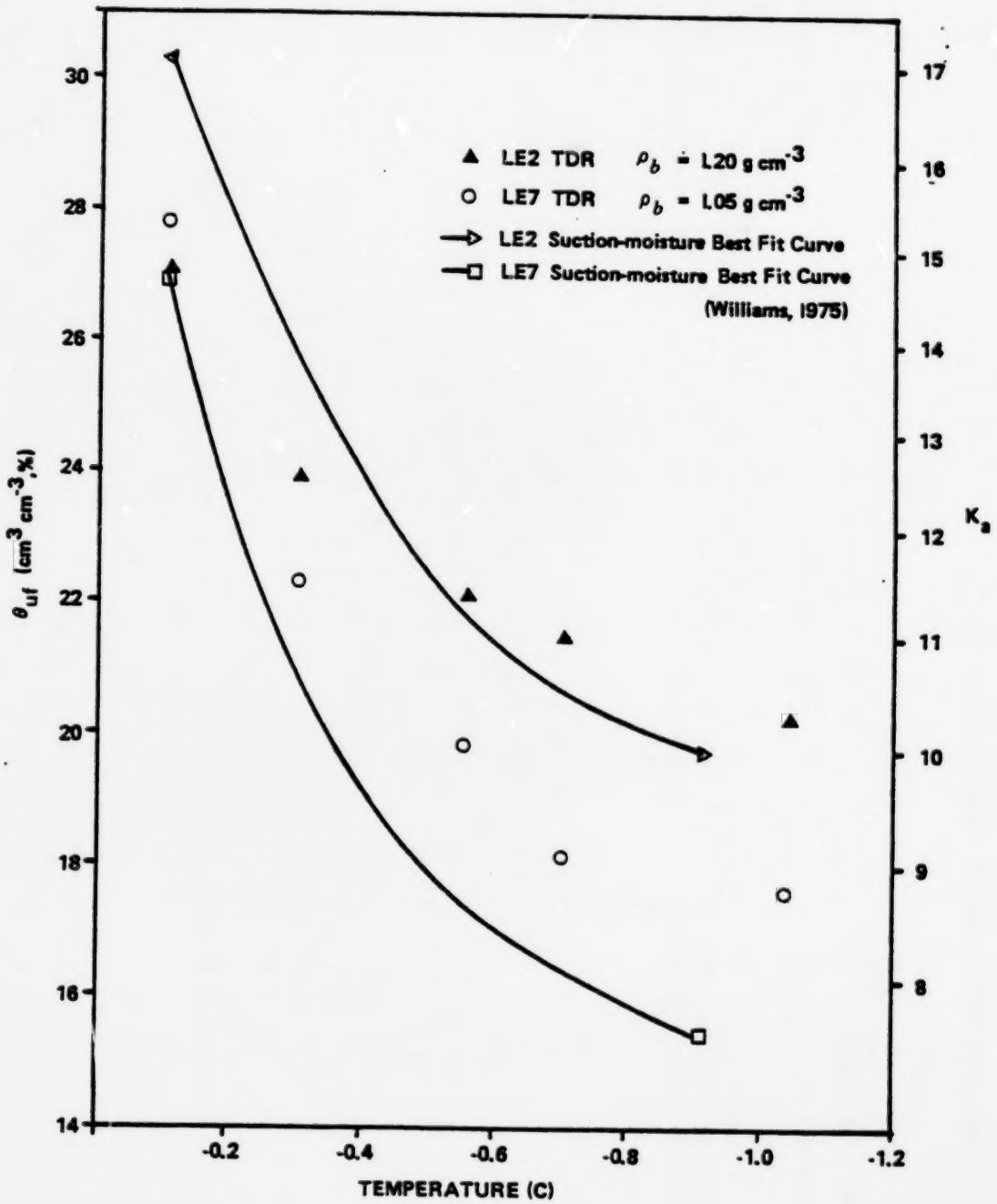
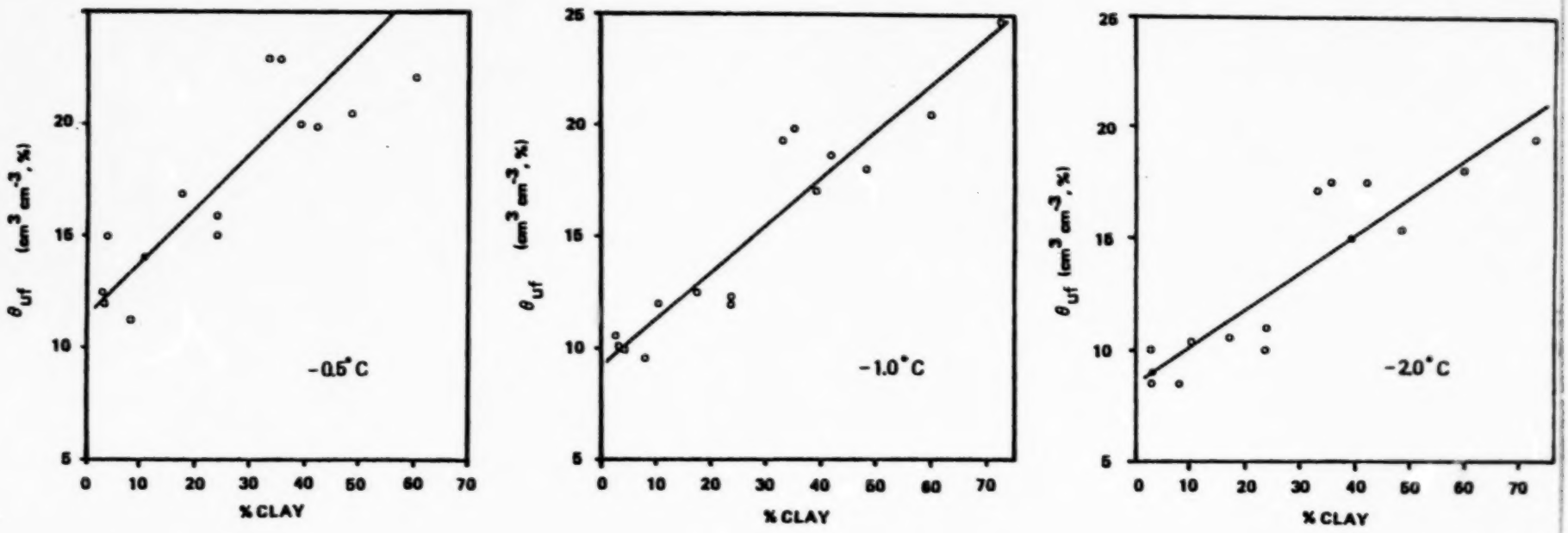


Figure 6 Freezing Characteristic Data: Norwegian Soils



θ_{uf} = 0.243(% < 2μ)
r = 0.91 S.E. = 1.02

θ_{uf} = 0.209(% < 2μ)
r = 0.94 S.E. = 0.55

θ_{uf} = 0.170(% < 2μ)
r = 0.91 S.E. = 0.71

Figure 7 Unfrozen Water Content Versus Clay Content

The results, shown in Figure 7, show a strong correlation between θ_{uf} and the percentage of soil particles finer than 2μ. Although the data are limited, they serve to emphasize the role that clay content has upon the soil freezing characteristic curve. However, any relationship could be affected by clay mineralogy (e.g., expanding clays) or extremes in soluble salt content.

Freezing characteristic data for Allendale silty clay were obtained for a variety of samples with dry bulk densities in the range 1.18 to 1.31 g cm^{-3} and at degrees of saturation from 66 to 96%. The test results are presented in Figure 8 (see also Appendix II). As shown, θ_{uf} at any given temperature falls within $\pm$2% for all samples, suggesting that such ranges of density variations and degree of saturation have a minimal effect upon the freezing characteristic curve. Similar results were also found for Garden sandy loam (see Appendix II). The effect of soil bulk density will be discussed further in Section 3.3, where results for low density samples are presented.

3.2 Freezing Column Experiments

The test results from two experiments are shown in Figures 9 and 10, and Tables 2 and 3. The data for Ellwood clay loam show that the θ_v profile was essentially uniform at 0 hours but displayed marked desiccation in the unfrozen zone after 122 hours, implying water movement from the unfrozen to the frozen zone during freezing. Gravimetric sampling of the column at 122 hours indicated an increase in total water content in the frozen section. Table 2 shows that the calculated water balance (i.e., water loss from the unfrozen zone vs. water gain in the

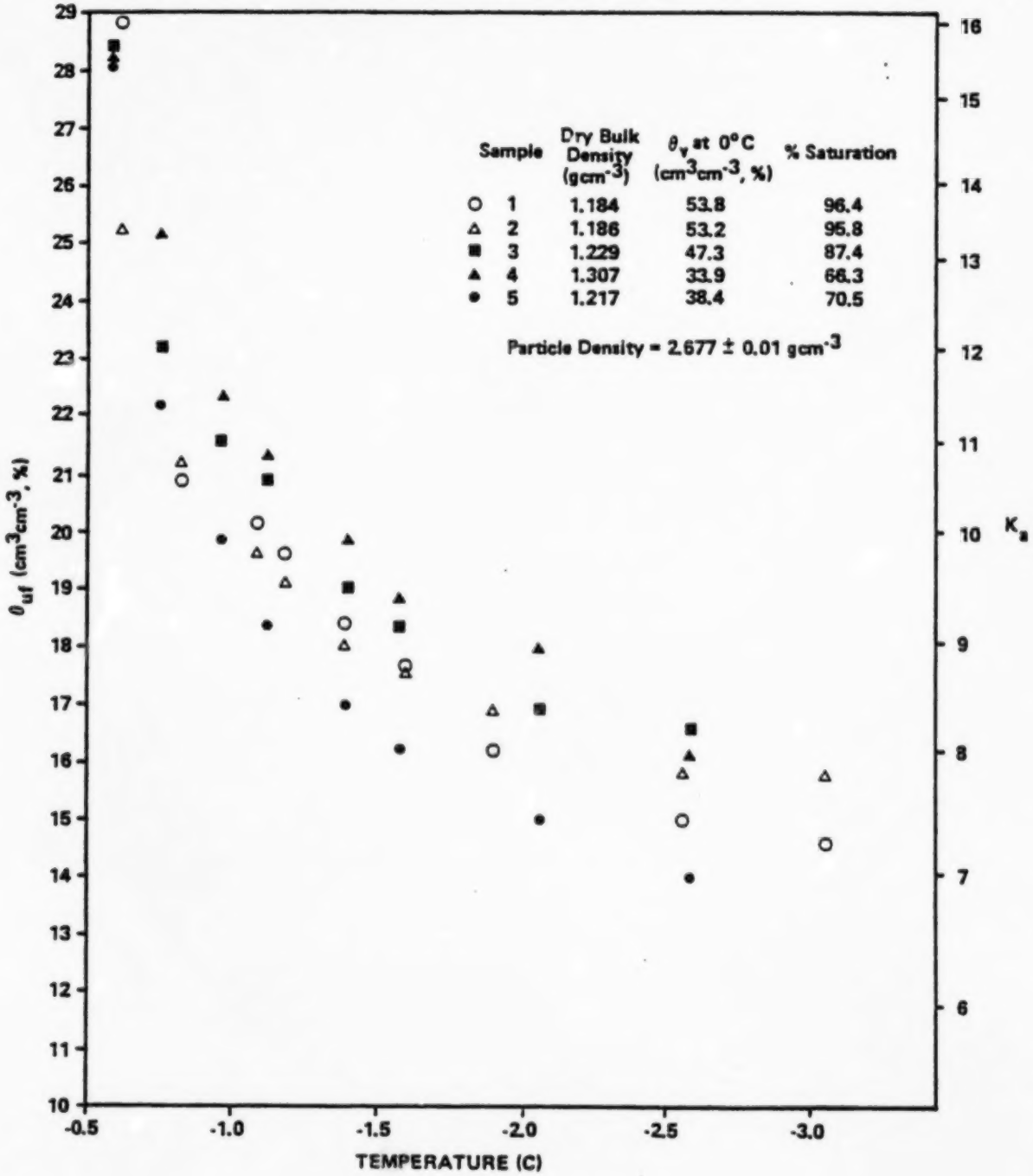


Figure 8 Freezing Characteristic Data for Allendale Silty Clay at Various Degrees of Saturation

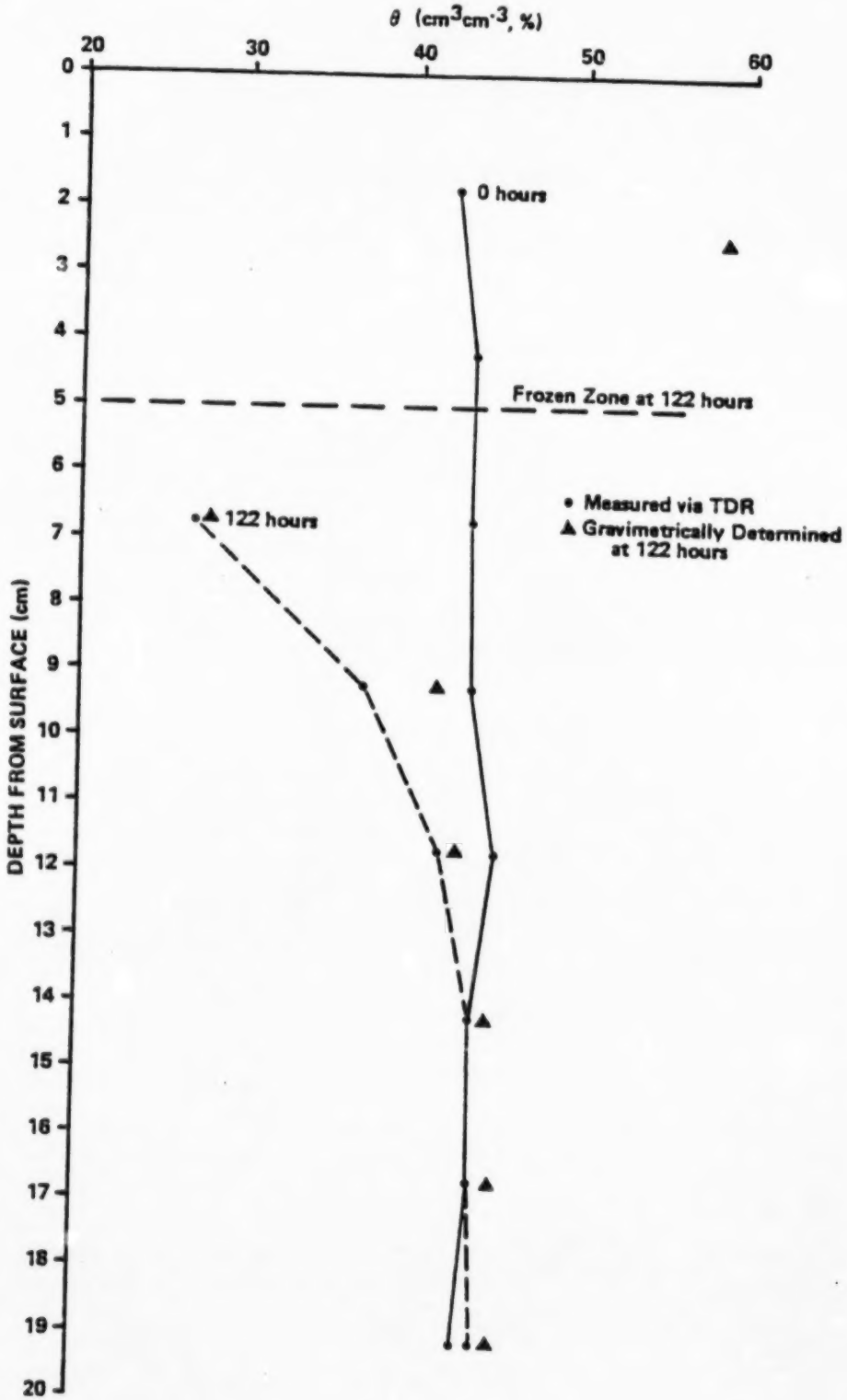


Figure 9 Results from Freezing Column Experiment: Ellwood Clay Loam

TABLE 2

FREEZING COLUMN RESULTS (ELLWOOD CLAY LOAM)

WATER CONTENT DATA

Soil Layer (cm)	θ (cm^3 cm^{-3}, %) by TDR		θ from gravimetric
	0 hr	122 hrs	122 hrs
0.5 - 3.0	42.5	Frozen	
3.0 - 5.5	43.7	Frozen	58.4 (0-5.0 cm)
5.5 - 8.0	43.7	26.8	27.2 (5.0-8.0 cm)
8.0 - 10.5	43.7	37.2	41.6
10.5 - 13.0	45.0	41.9	42.5
13.0 - 15.5	43.7	43.7	44.2
15.5 - 18.0	43.7	43.7	44.7
18.0 - 20.5	42.8	43.7	44.7

WATER BALANCE DATA

Volume of H_2O (cm^3)

Soil Layer (cm)	(1) 0 hr TDR	(2) 122 hrs TDR	(3) 122 hrs gravimetre	(4) mean of (2) and (3)	Change in H_2O Volume (4)-(1)
0- 5.0	290.5	Frozen	393.6	393.6	+ 103.1
5.0- 8.0	176.7	108.4	109.9	109.2	- 67.5
8.0-10.5	147.2	127.4	140.2	133.8	- 13.4
10.5-13.0	151.6	141.2	143.2	142.2	- 9.4
13.0-15.5	147.2	147.2	148.9	148.1	-
13.5-18.0	147.2	147.2	150.6	148.9	-
18.0-20.5	144.2	147.2	150.6	148.9	-
	Total H_2O = 1204.6		Total H_2O = 1237.0		

Volume of H_2O moved into frozen zone = 103 cm^3
Volume of H_2O moved out of unfrozen zone = 90.3 cm^3

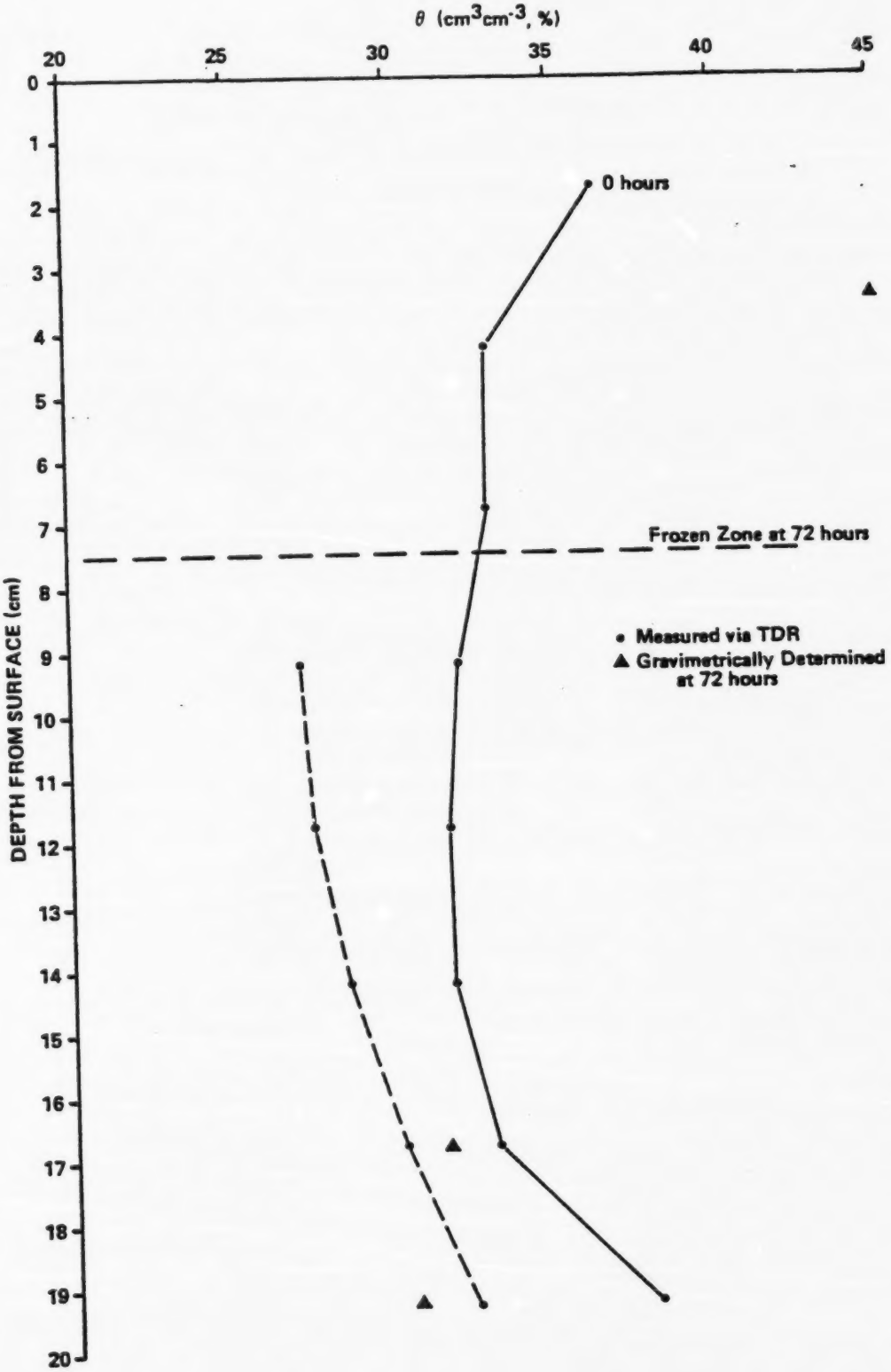


Figure 10 Results from Freezing Column Experiment: Cavan Loam

TABLE 3

FREEZING COLUMN RESULTS (CAVAN LOAM)

WATER CONTENT DATA

Soil Layer (cm)	θ (cm^3 cm^{-3}, %) by TDR		θ from gravimetric
	0 hr	72 hrs	72 hrs
0.5 - 3.0	36.4	Frozen	
-3.0 - 5.5	33.0	Frozen	
5.5 - 8.0	33.0	Frozen	45.0 (0-7.5 cm)
8.0 - 10.5	32.0	27.2	Unable
10.5 - 13.0	31.8	27.4	to
13.0 - 15.5	31.8	28.5	sample
13.5 - 18.0	33.0	30.1	31.6
18.0 - 20.5	38.9	32.2	30.5

WATER BALANCE DATA

Soil Layer	Vol. H_2O at 0 hrs	Vol. H_2O at 72 hrs	Change in H_2O Volume
0 - 7.5	350.7	454.8	104.1
7.5 - 10.5	129.4	109.9	-19.5
10.5 - 13.0	107.1	92.3	-14.8
13.0 - 15.5	107.1	96.0	-11.1
15.5 - 18.0	111.2	101.4	- 9.8
18.0 - 20.5	131.1	108.5	-22.6

Volume of H_2O moved into frozen zone = 104.1
Volume of H_2O moved out of unfrozen zone = 77.8

frozen zone) is in good agreement. It should also be noted that the TDR and gravimetric measurements of θ_v at 122 hours in the unfrozen zone were in good agreement.

The Cavan loam results also show desiccation of the unfrozen zone and an increase in water content in the frozen portion. The calculated water balance agrees fairly well although the agreement is not as good as that for the Ellwood sample, presumably because of larger experimental errors in the gravimetric sampling.

Although the results are preliminary in nature, they seem to indicate that the TDR technique can be a useful tool for monitoring the desiccation of the unfrozen zone in closed system soil freezing. Further experimentation is required to substantiate these preliminary findings. Unfortunately, in neither of these first experiments can anything definite be said about the changes in K_a over time in the frozen zone. This is because, with a spacing of 2.5 cm, the field of the probe overlapped both the frozen and unfrozen zones. For example, in the Ellwood sample a marked decrease in K_a was measured by the probe just behind the frost front; however, it is not possible to say whether this is due to excess ice accumulation in the frozen zone (see Section 3.3), desiccation in the unfrozen zone, or a combination of both effects.

A new freezing column has closer-spaced probes (1.5 cm), and closer attention will be paid to ensure that the probe just behind the freezing front is completely within the frozen zone. Thus we can monitor the effect of ice accumulation on K_a, and then be able to assess the use of the TDR technique to determine areas of excess ice accumulation in the frozen zone and desiccation of the unfrozen zone.

3.3 Freezing Characteristic Data for Samples with Excess Ice

The effect of dry bulk density (ρ_b) on the soil freezing characteristic curve was examined for densities which are representative of possible field values (see Section 3.1). However, when a soil freezes and water migration occurs, zones of excess ice form, effectively decreasing ρ_b. Although we cannot use the TDR technique to measure ice content directly, increases in ice content, by ice lensing, should result in a decrease in the measured K_a due to the displacement of soil and unfrozen water. By examining freezing characteristic curves for low density samples (generally less than 1 g cm^{-3}) we have tried to assess whether the TDR technique could be used to delineate areas of excess ice formation in freezing soils. If successful, it could be used in situ or in laboratory freezing soil columns.

A number of authors have suggested that the gravimetric unfrozen water content, w (gg^{-1}, %), should not be affected by the ice content of the soil (e.g., Williams, 1967; Anderson and Morgenstern, 1973). If so, then θ_{uf} is directly proportional to ρ_b as follows:

$$\theta_{uf} = \frac{w \cdot \rho_b}{\rho_w} \qquad (1)$$

where ρ_b is the density of water (1 g cm^{-3}). Thus, as ρ_b decreases, so does θ_{uf}, and so should the measured value of K_a. These relationships are illustrated, hypothetically, in Figures 11 and 12. Starting with an assumed freezing characteristic curve, on a gravimetric basis (w vs. T°C), this was converted to a volumetric basis, for various bulk densities, using equation (1). The K_a scale for Figure 12 was derived from the relationship between K_a and θuf tabulated in Appendix I.

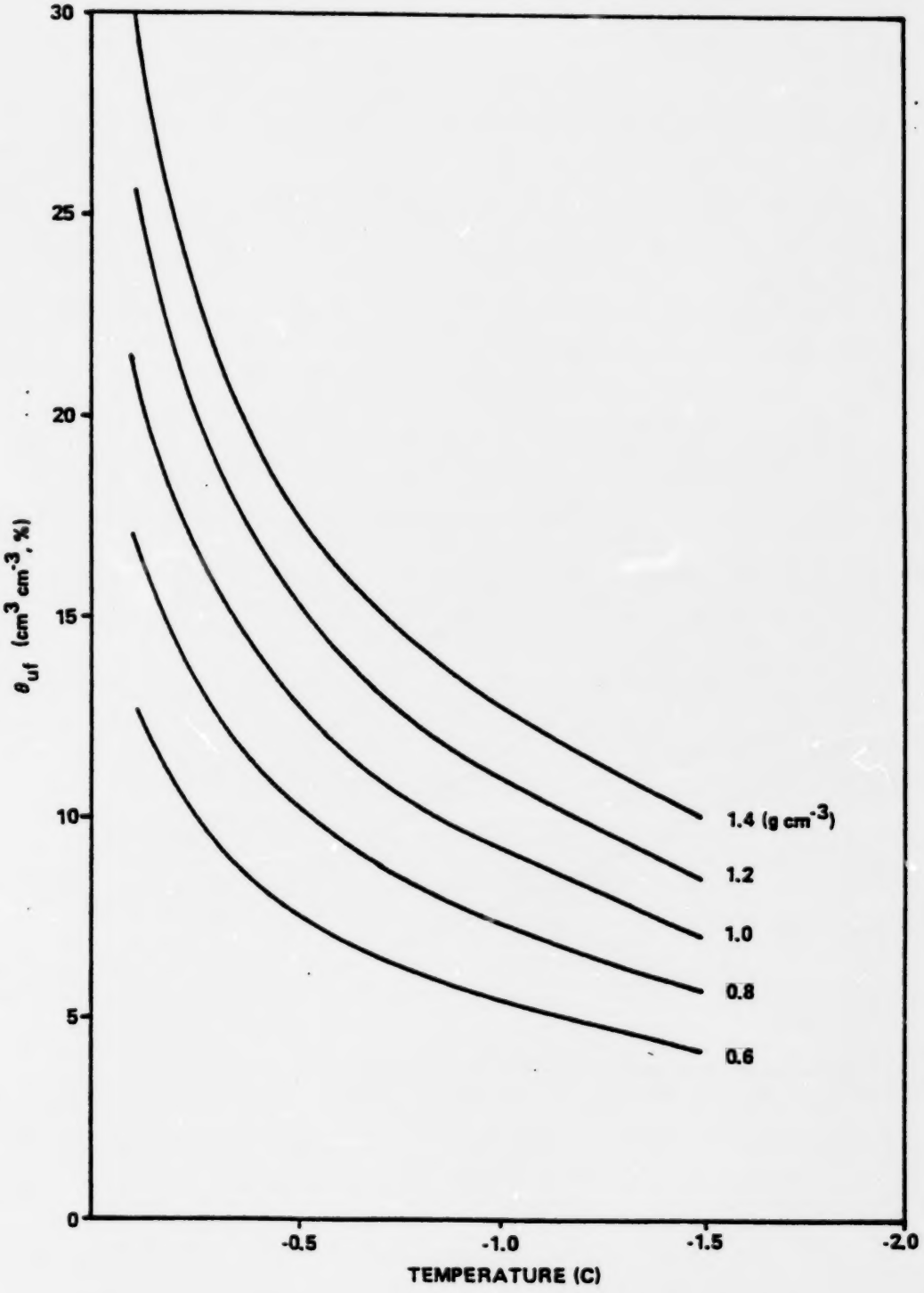


Figure 11 Theoretical Freezing Characteristic Curves for Various Dry Bulk Densities

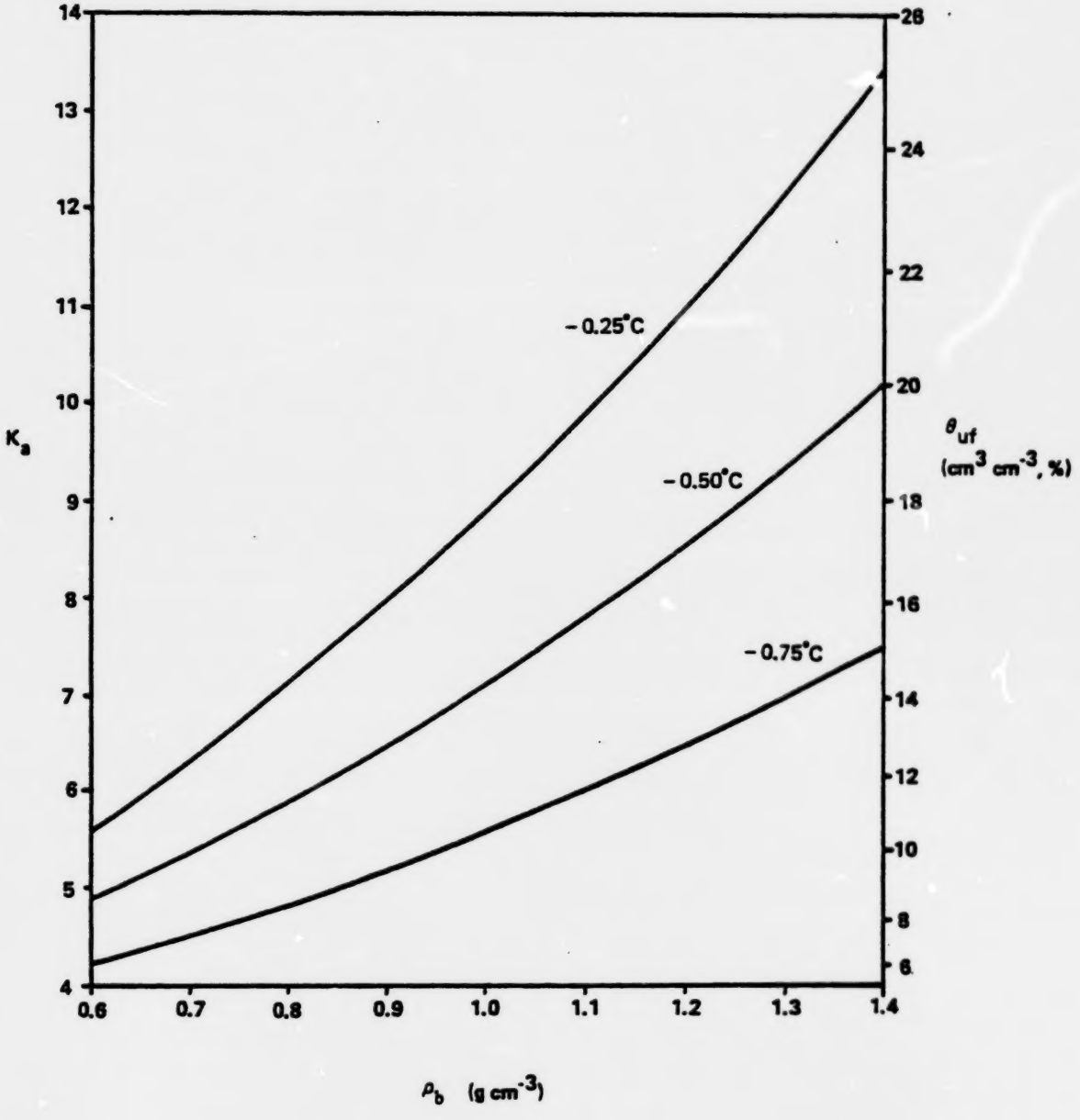


Figure 12 Theoretical Changes in K_a with Dry Bulk Density at Three Temperatures

Since θ_{uf} decreases as ρ_b decreases, it is expected that measured values of K_a for samples containing excess ice will be less than K_a for the same soil at normal density. i.e.,

i) If K_a, measured at a certain temperature, is less than the K_a known for the same soil at normal density, then ρ_b is lower, for example, because of excess ice (see Figure 11).

ii) If, at a certain temperature, K_a is observed to decrease over time, this indicates a decrease in ρ_b (Figure 12), for example, because of excess ice formation.

To examine these ideas, K_a vs. T°C data were determined for Manchester silt, Castor sandy loam and Allendale silty clay at a variety of bulk densities (i.e., excess ice contents). The results have been converted into unfrozen water contents, using the relationship in Appendix I for θ_{uf} and equation (1) for w; they are shown in Figures 13 to 18. The experimental results confirm that:

i) At a given temperature, θ_{uf} (K_a) decreases as ρ_b decreases (Figures 13, 15, and 17).

ii) At a given temperature, w is "constant" (within $\pm$ 2%) regardless of the ice content (Figures 14, 16, and 18).

To take a specific example, we can look at the effect of excess ice content on K_a for Allendale silty clay (Figure 17). At a temperature of -0.7°C, we have the following data at various bulk densities:

ρ_b (g cm^{-3})	K_a	θ_{uf} (cm^3 cm^{-3}, %)	θ_i (cm^3 cm^{-3}, %)*
1.186	9.88	19.6	34.2
0.825	7.84	15.8	43.7
0.683	6.76	13.4	56.6
0.545	5.14	9.4	57.2

*Ice content determined gravimetrically

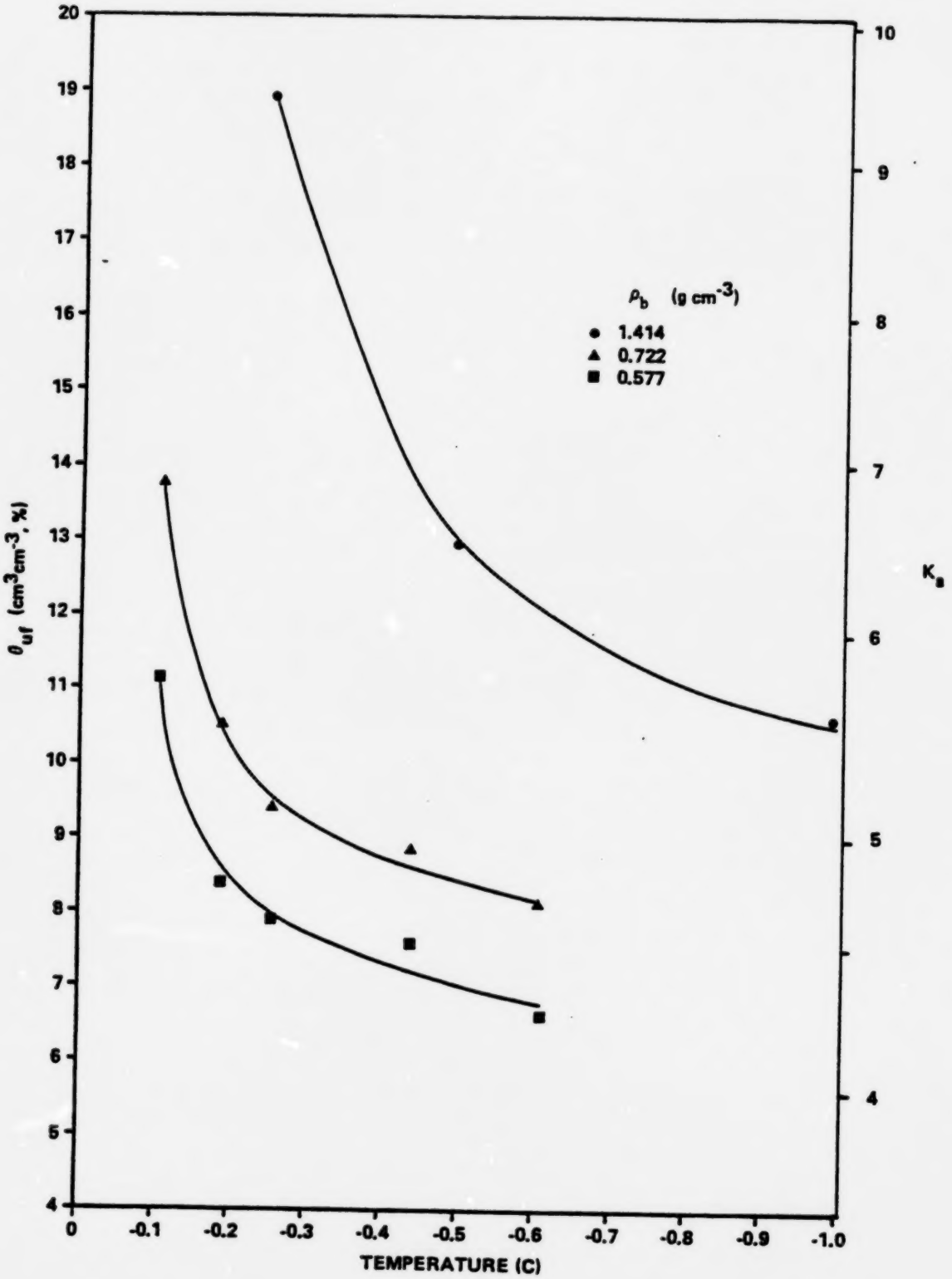


Figure 13 Variation in Freezing Characteristic Curve with Bulk Density (Manchester Silt)

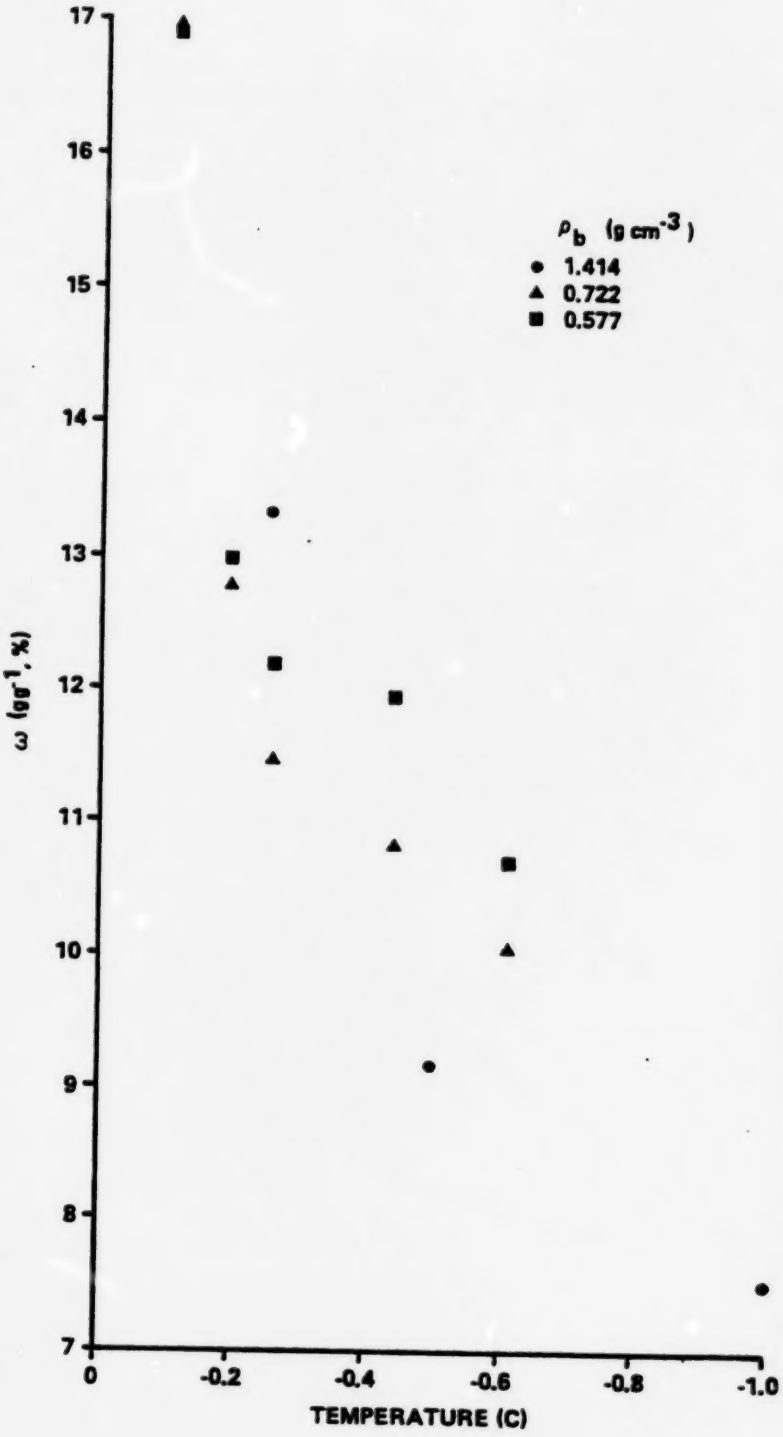


Figure 14 W-T°C for Manchester Silt

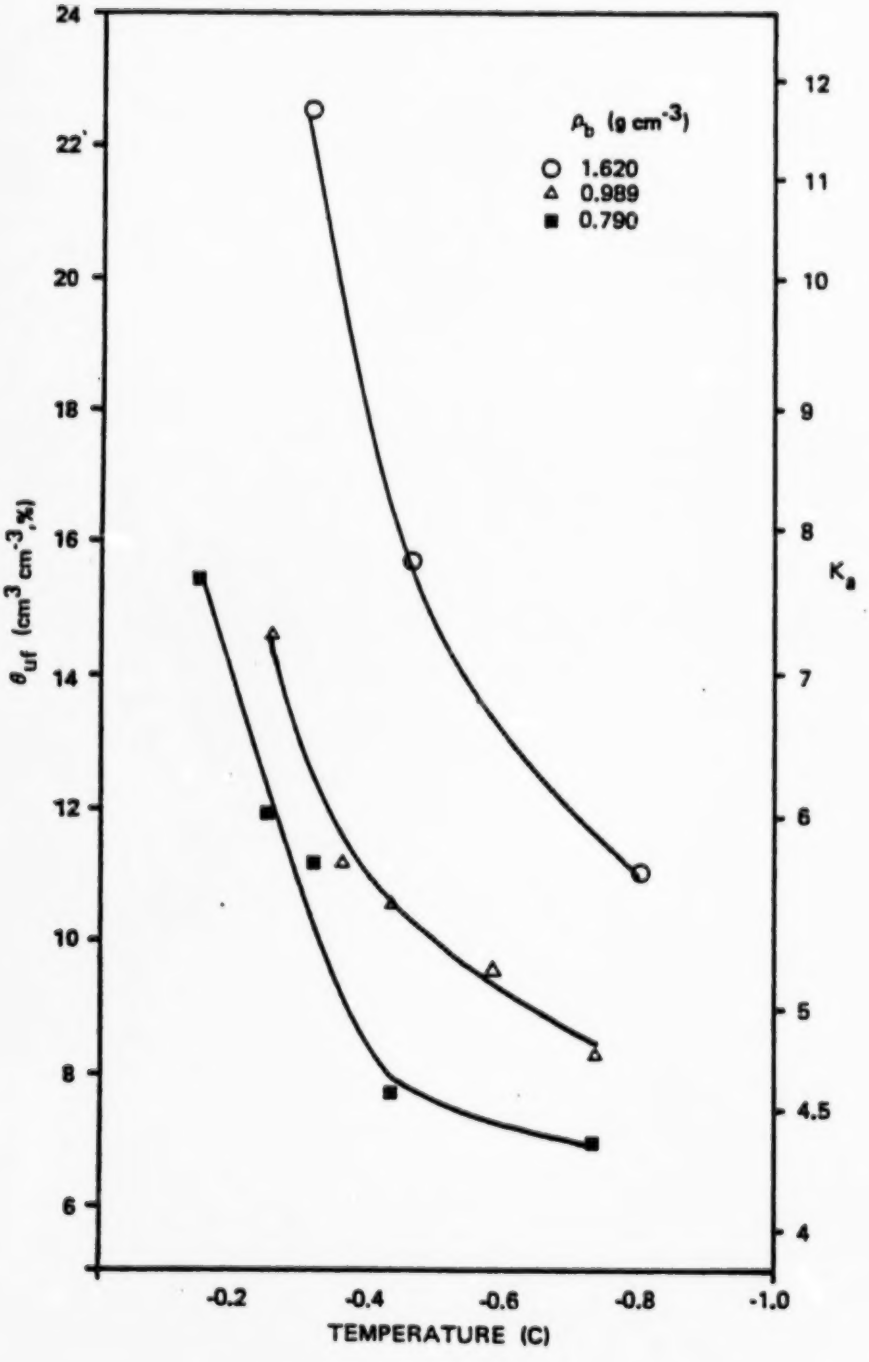


Figure 15 Variation in Freezing Characteristic Curve with Bulk Density (Castor Sandy Loam)

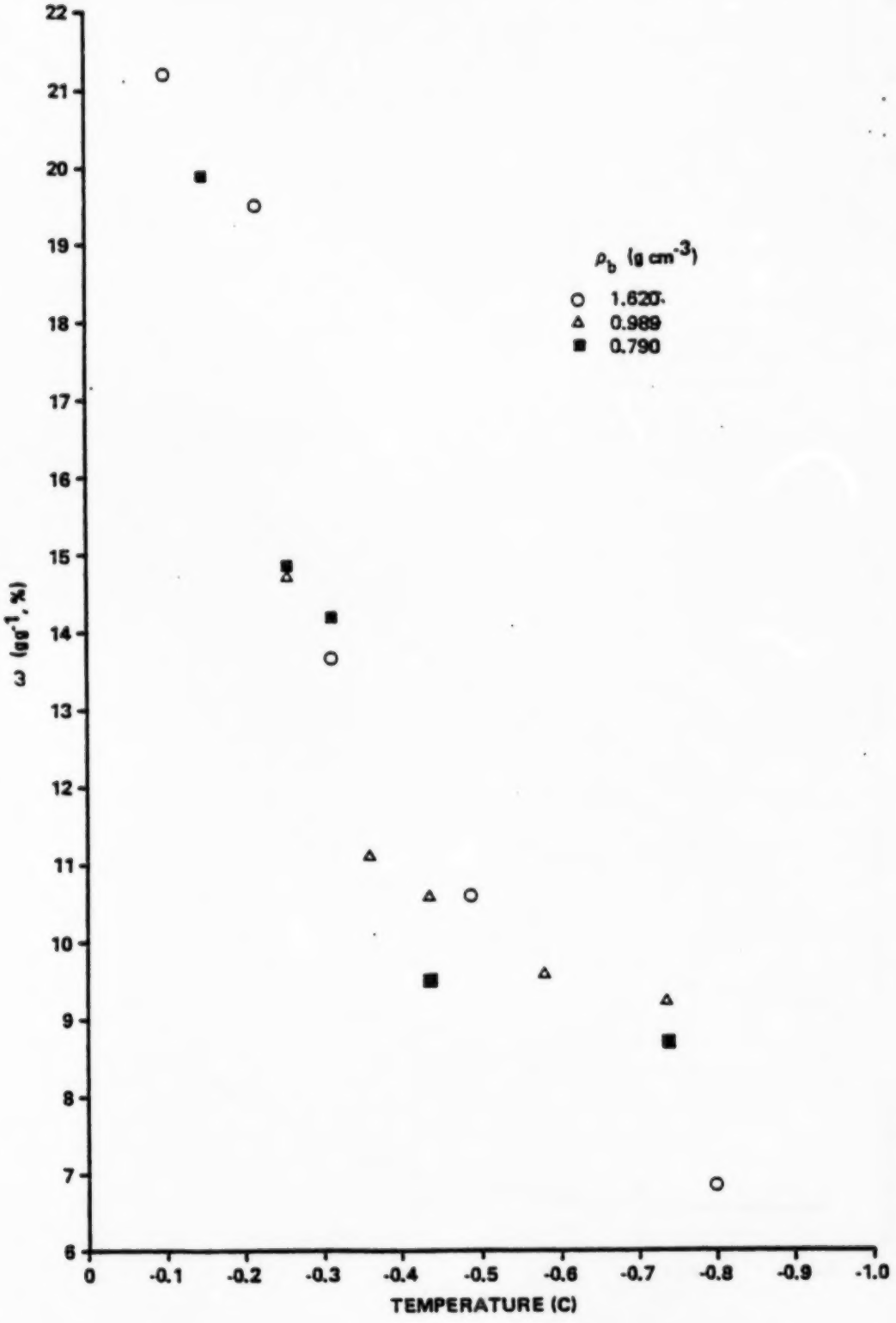


Figure 16 w - T °C for Castor Sandy Loam

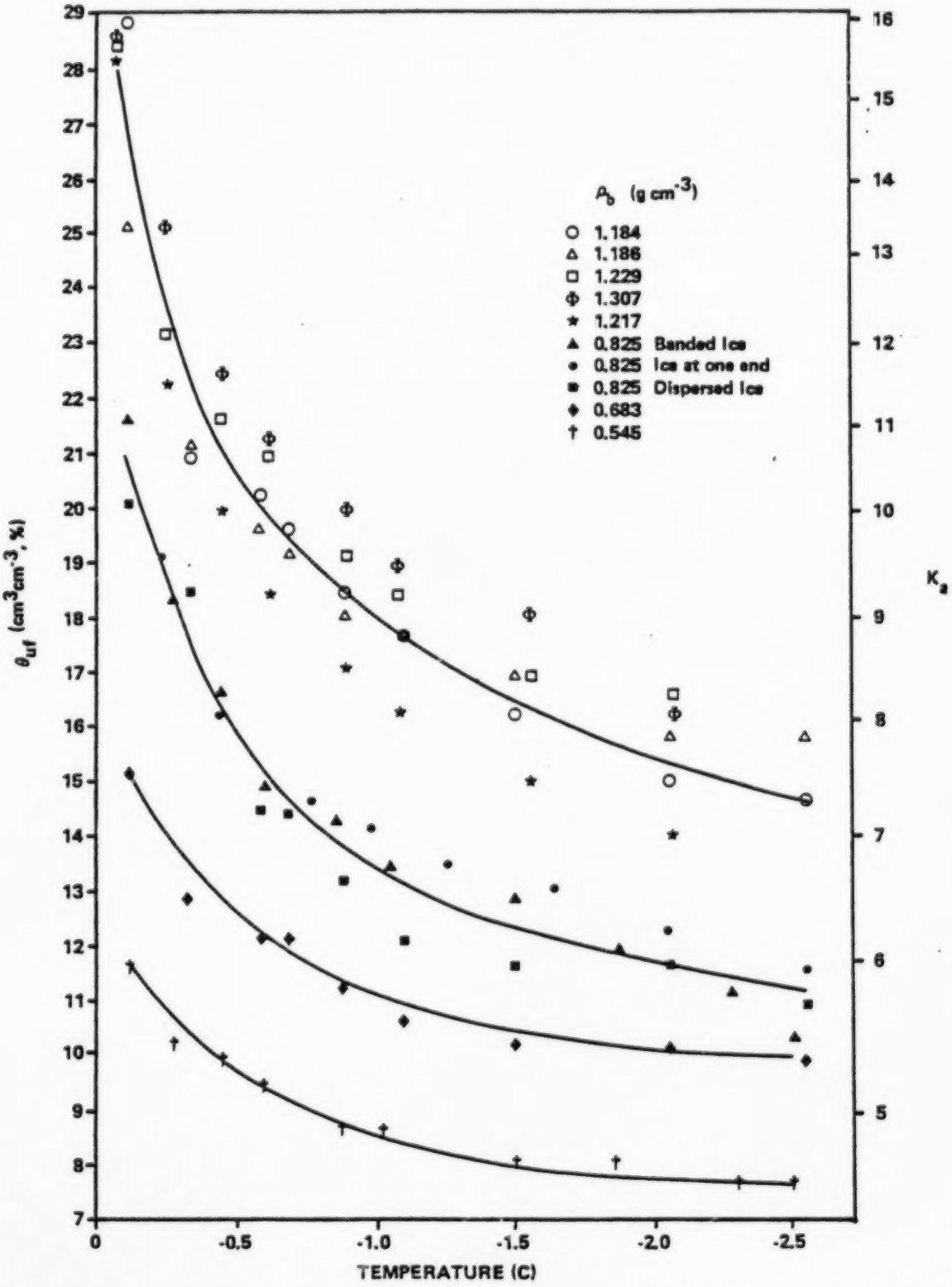


Figure 17 Variation in Freezing Characteristic Curve with Bulk Density (Allendale Silty Clay)

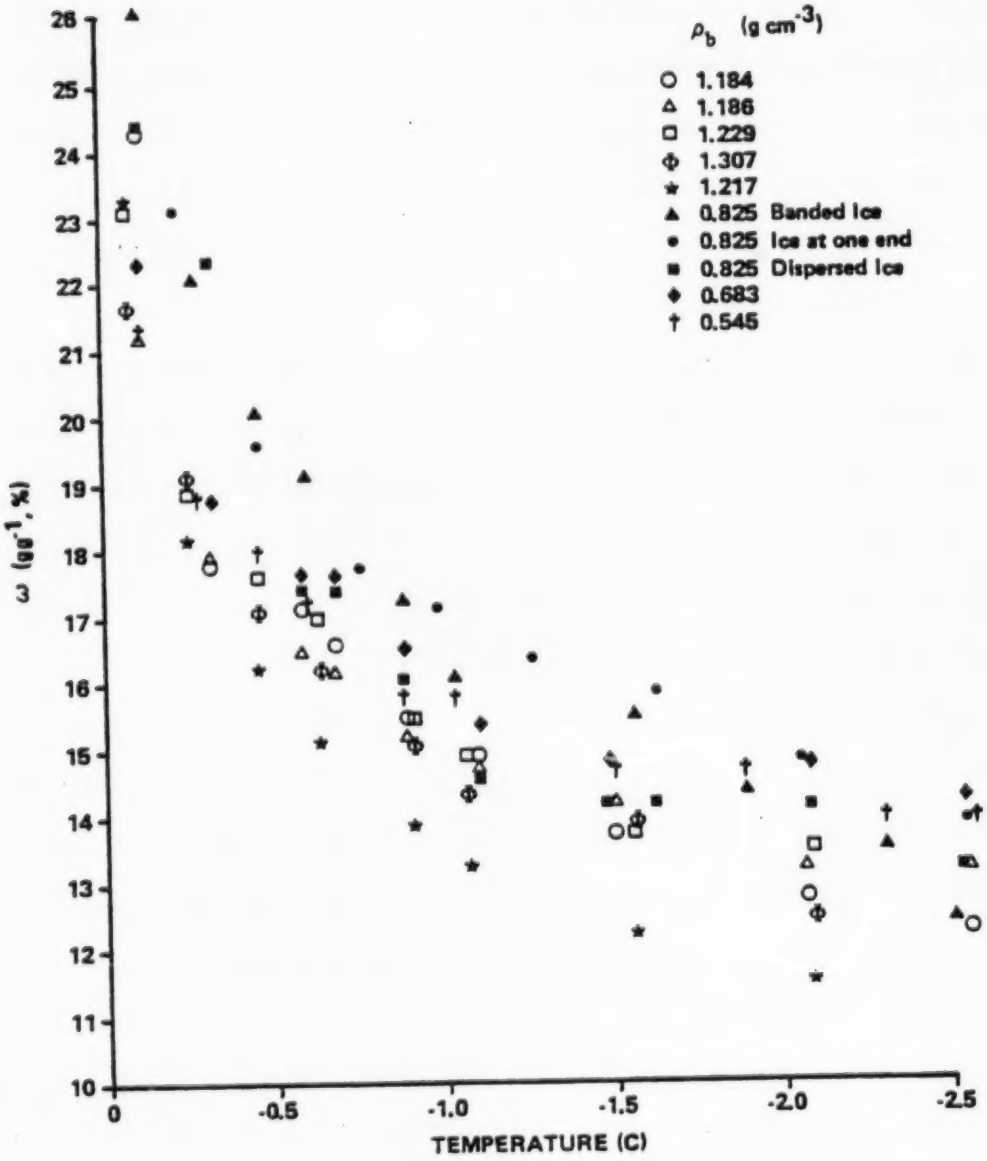


Figure 18 W-T°C for Allendale Silty Clay

Thus there appears to be some evidence for a relationship between K_a and θ_i and in view of the above results it might be possible to infer an ice content from TDR measurement of K_a. However, since the TDR technique cannot measure ice content directly, the more term of "maximum possible ice content" is used in the following discussion. A nomogram has been prepared showing the relationship between maximum possible ice content, measured. θ_{uf} (K_a) and bulk density (Figure 19).[1] The nomogram assumes that w is constant (at any temperature) and that the soil contains no air. This results in a maximum possible ice content. Actual values may be lower than this if air is present (see below). The only information required are the freezing characteristic data at "normal" bulk density, and the measured K_a (θ_{uf}) at some known temperature for the sample in question. For example, Figure 19 shows that a soil at a given temperature below 0°C has a θ_{uf} of 21% and a dry bulk density of 1.4 g cm^{-3}; therefore w is 15% (Point A) and the maximum possible ice content is about 28.5% (Point B). If a sample later shows a θ_{uf} of 10% (at the same temperature), ρ_b is about 0.63 g cm^{-3} (Point C, assuming constant w) and the maximum possible ice content is about 65% (Point D). Applying the same logic to the example of the Allendale silty clay, we get the following results:

ρ_b	θ_i (Actual)	θ_i (Nomogram)
1.186	34.2	35.8
0.825	43.7	55.2
0.683	56.6	66.2
0.545	57.2	70.0

[1] A full description of the nomogram is given in Appendix III.

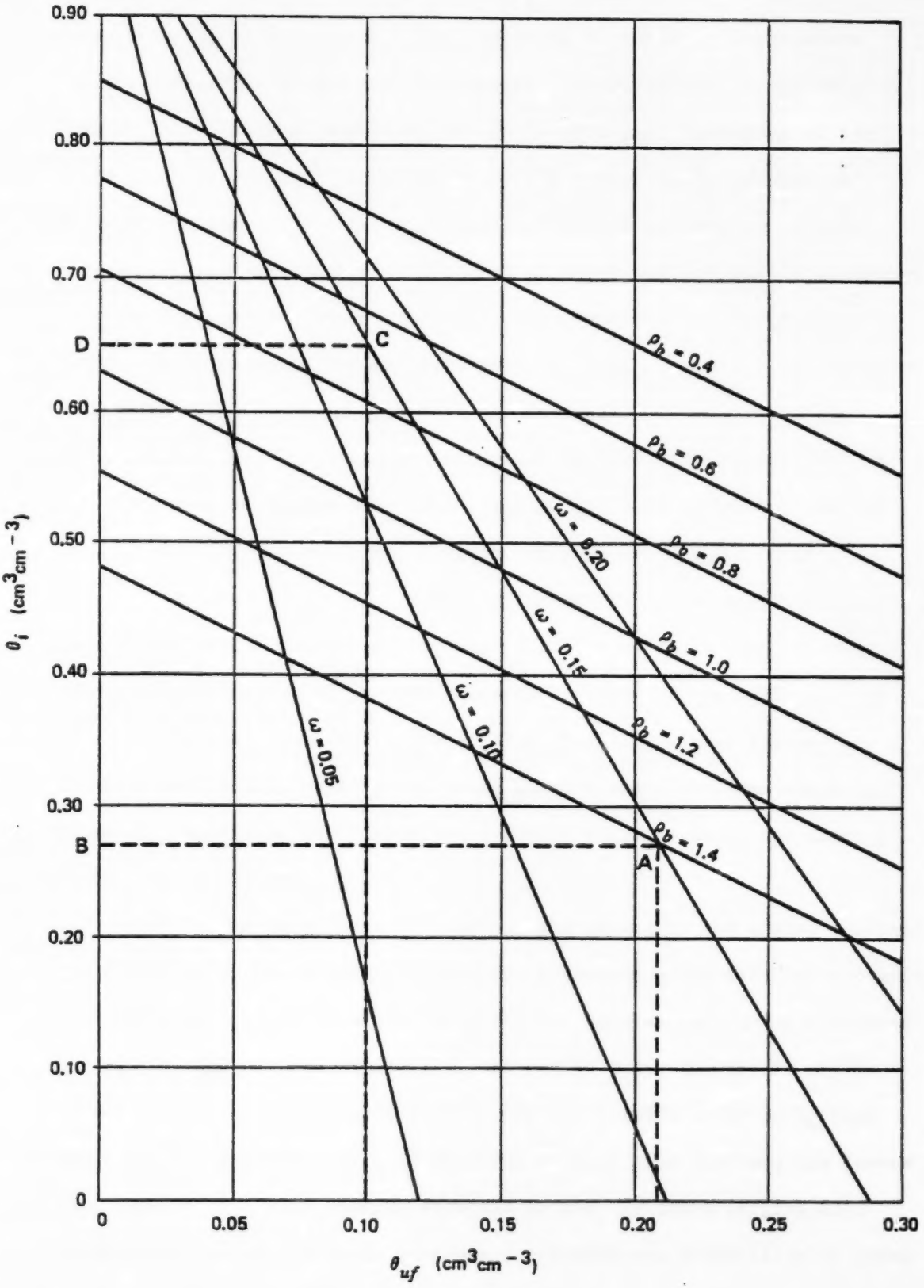


Figure 19 Nomogram for Inferring Maximum Ice Content from Measured θ_{uf}

Apart from the first sample (which contained virtually no air), there is some difference in the ice contents as a result of air in the samples. However, the inferred (nomogram) values do give some indication of the build-up of excess ice. Unfortunately, one cannot really improve on the nomogram since the TDR cannot resolve between air and ice in soil. Figure 20 (from Smith and Patterson, 1980) shows only a very slight change in K_a as ice replaces air in the soil, and certainly not enough of a difference to be exploited in the present context. Therefore we can only talk in terms of a maximum possible ice contect.

However, the potential exists for using TDR measurements of K_a to identify zones of excess ice. For example, ice content can be inferred if K_a changes over time whilst temperature does not change. Such conditions can occur in a freezing soil column where temperature can be readily controlled and a water source (from the unfrozen zone) is available. The usefulness of the TDR approach for identifying areas of ice accumulation will have to be investigated further via open system freezing soil column experiments using large diameter columns which can be readily sectioned to independently obtain water contents and density changes in the frozen section.

Finally, we have looked at whether the nature of the excess ice has any effect on K_a measurements. Data for Allendale silty clay, at a density of 0.825 g cm^{-3}, were obtained for different samples containing dispersed ice and ice lenses (see Figure 17). As can be seen, the nature of the excess ice has no appreciable effect upon the overall value of K_a (and hence (θ_{uf}). However, it is interesting to note that discrete ice lenses in a coaxial line were readily detected on the TDR trace (Figure 21). Experiments are currently in progress to examine the possibility of using

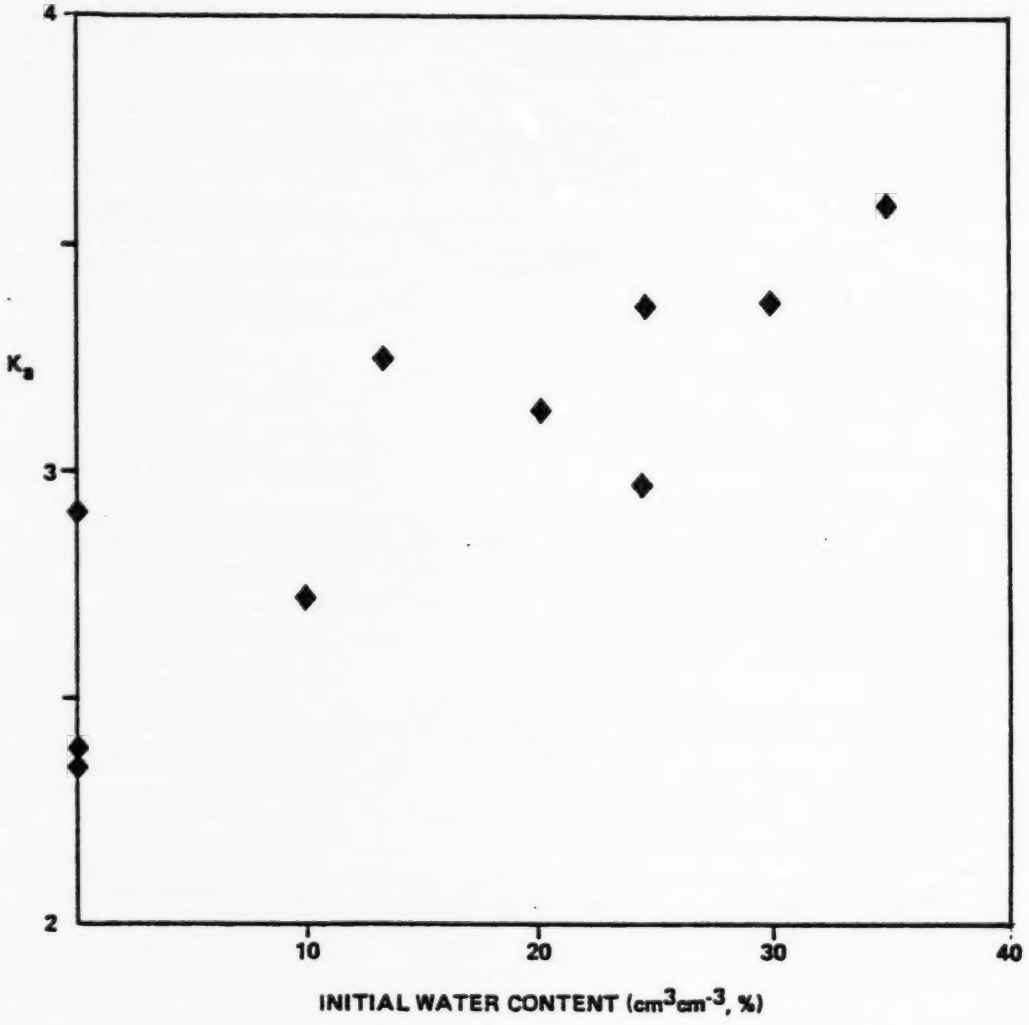


Figure 20 K_a for Frozen Ottawa Sand at Different Initial Water Contents

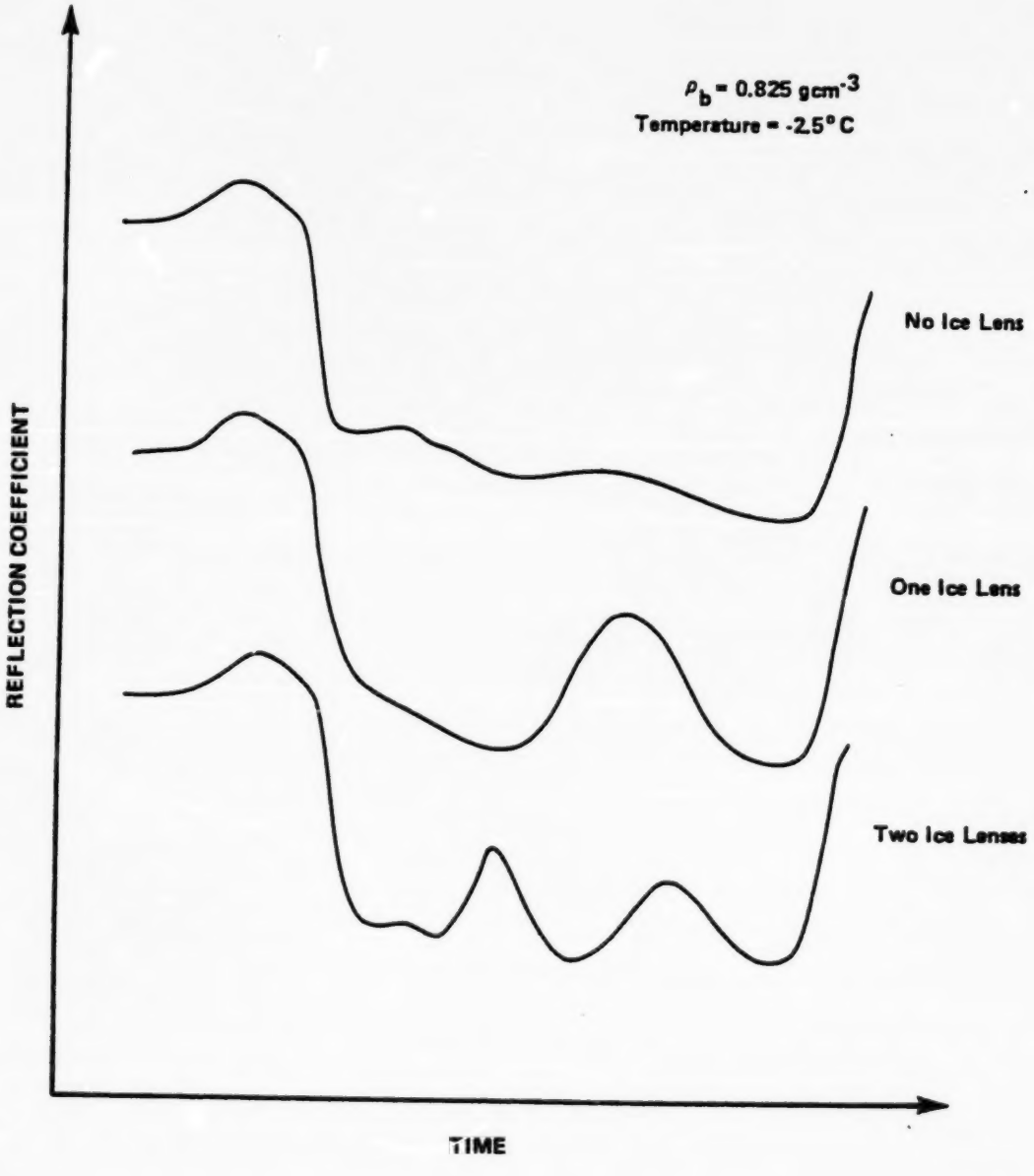


Figure 21 TDR Traces for Allendale Silty Clay with Excess Ice

parallel transmission lines to identify zones of excess ice accumulation in freezing soils. The ability to identify ice lense growth would have many practical applications, such as round refrigerated pipelines or beneath highways.

SECTION 4

CONCLUSIONS

The TDR technique now shows great promise for routine measurement of volumetric liquid water contents in frozen soils as suggested by Smith and Patterson (1980). The extension of the TDR work to samples containing excess ice and to instrumenting soil freezing columns suggests that the TDR technique may prove to be a valuable non-destructive laboratory technique for obtaining hydrologic data for soil freezing experiments.

Specific findings from the experiments reported here include:

1. Values of θ_{uf} determined via the TDR technique and suction-moisture tests agree to within $\pm$ 2.5% in θ_{uf}.
2. Freezing characteristic data obtained from samples of various field densities and degrees of saturation agree well ($\pm$ 2% in θ_{uf}).
3. A relationship between clay content and θ_{uf}, at various temperatures was found for soils tested via the TDR technique.
4. The unfrozen water content, expressed gravimetrically, for a soil at a given temperature is within $\pm$ 2% regardless of ice content.
5. K_a, at a given temperature, is less in samples containing excess ice (dry bulk densities less than 1 g cm^{-3}) than for samples without excess ice. A method has been devised for inferring a maximum possible ice content by comparing a measured value of K_a (θ_{uf}) with that expected for the same soil at normal field density (i.e., from the normal freezing characteristic curve).

6. Preliminary freezing column experiments indicate that the TDR technique may be a viable method of determining the rates and amount of desiccation in the unfrozen zone during closed system soil freezing.

Continued research should be undertaken to examine the θ_{uf} -T °C relationship for soil samples containing excess ice since these data will help to assess the use of the TDR technique to delineate zones of ice lens growth in freezing soils columns or perhaps even in situ. Experiments are currently being undertaken to examine the potential use of balanced parallel transmission lines to locate zones of excess ice in soil profiles. Continued freezing soil column experimentation is needed to examine the use of the TDR technique as a non-destructive method of providing data on heat and mass transfer in freezing soils. Field sites should also be instrumented to assess the use of the TDR technique for monitoring water movement and ice segregation in natural freezing ground.

ACKNOWLEDGEMENTS

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Appendix I

TABULATION OF THE EMPIRICAL RELATIONSHIP GIVEN BY TOPP, DAVIS AND ANNAN (1980)

K_a	Vol. Water Cont. (decimal fraction)	K_a	Vol. Water Cont. (decimal fraction)
3.03	0.000	11.64	0.225
3.08	0.005	11.96	0.230
3.14	0.010	12.28	0.235
3.20	0.015	12.61	0.240
3.27	0.020	12.94	0.245
3.35	0.025	13.28	0.250
3.44	0.030	13.62	0.255
3.53	0.035	13.97	0.260
3.63	0.040	14.32	0.265
3.74	0.045	14.67	0.270
3.85	0.050	15.03	0.275
3.97	0.055	15.40	0.280
4.10	0.060	15.76	0.285
4.23	0.065	16.13	0.290
4.37	0.070	16.51	0.295
4.52	0.075	16.89	0.300
4.67	0.080	17.27	0.305
4.83	0.085	17.66	0.310
4.99	0.090	18.05	0.315
5.17	0.095	18.44	0.320
5.34	0.100	18.84	0.325
5.53	0.105	19.24	0.330
5.72	0.110	19.65	0.335
5.91	0.115	20.05	0.340
6.12	0.120	20.47	0.345
6.32	0.125	20.88	0.350
6.54	0.130	21.30	0.355
6.76	0.135	21.72	0.360
6.98	0.140	22.15	0.365
7.21	0.145	22.57	0.370
7.45	0.150	23.00	0.375
7.69	0.155	23.44	0.380
7.94	0.160	23.87	0.385
8.19	0.165	24.31	0.390
8.45	0.170	24.76	0.395
8.72	0.175	25.20	0.400
8.99	0.180	25.65	0.405
9.26	0.185	26.10	0.410
9.54	0.190	26.55	0.415
9.83	0.195	27.01	0.420
10.12	0.200	27.47	0.425
10.41	0.205	27.93	0.430
10.71	0.210	28.39	0.435
11.02	0.215	28.85	0.440
11.33	0.220	29.32	0.445

Appendix I Continued

K_a	Vol. Water Cont. (decimal fraction)	K_a	Vol. Water Cont. (decimal fraction)
29.79	0.450	32.65	0.480
30.26	0.455	33.13	0.485
30.74	0.460	33.62	0.490
31.21	0.465	34.10	0.495
31.69	0.470	34.59	0.500
32.17	0.475	35.08	0.505

APPENDIX II

FREEZING CHARACTERISTIC DATA

θ_{uf} is the volumetric water content ($cm^3\ cm^{-3}$, %).
K_a is the apparent dielectric constant.
ρ_b is the dry bulk density in g cm^{-3}.
$\bar{\theta}_{uf}$ is the mean θ_{uf}.

CASTOR SANDY LOAM (freeze cycle)

Sample	(1)		(2)		(3)		
°C	K_a	θ_{uf}	K_a	θ_{uf}	K_a	θ_{uf}	$\bar{\theta}_{uf}$
0	26.0	41.0	27.6	42.6	24.3	39.0	40.9
-0.20	8.31	16.8	9.40	18.8	9.00	18.0	17.9
-0.52	5.84	11.4	6.25	12.4	5.84	11.4	11.7
-0.70	5.76	11.1	5.60	10.7	5.68	10.9	10.9
-1.20	5.60	10.7	5.29	9.8	5.29	9.8	10.1
-1.85	5.29	9.8	[illegible]91	8.8	5.21	9.6	9.4
-2.50	4.69	8.0	4.55	7.6	4.91	8.8	8.1
-3.30	4.41	7.2	4.55	7.6	4.91	8.8	7.9
From gravimetric sampling:							
ρ_b		1.56		1.54		1.54	
θ at 0°C		39.2		39.2		39.2	

CASTOR SANDY LOAM <74μ (freeze cycle)

Sample	(1)		(2)		(3)		
°C	K_a	θ_{uf}	K_a	θ_{uf}	K_a	θ_{uf}	$\bar{\theta}_{uf}$
0	25.0	39.8	25.1	39.9	25.3	40.2	40.0
-0.10	19.5	33.3	20.9	35.0	20.7	34.8	34.4
-0.31	10.9	21.2	11.1	21.6	12.5	23.8	22.2
-0.46	7.47	15.0	8.02	16.2	7.84	15.8	15.7
-0.80	5.44	10.3	6.08	11.7	5.60	10.7	10.9
-1.60	4.69	8.1	5.44	10.2	4.98	9.0	9.1
-2.45	4.55	7.5	5.14	9.4	4.48	7.3	8.1
-3.45	4.20	6.4	4.62	7.8	4.34	6.8	7.0

From gravimetric sampling:

ρ_b	1.62	1.58	1.65
θ at 0°C	38.4	39.2	39.0

FIELD LOAMY SAND (freeze cycle)

°C	K_a	θ_{uf}
0	17.1	30.2
-0.15	7.65	15.4
-0.30	6.25	12.4
-0.55	5.60	10.6
-0.78	5.60	10.6
-1.54	4.84	8.6
-2.88	4.84	8.6
-3.94	4.55	7.6

From gravimetric sampling:

ρ_b	1.72
θ at 0°	30.4

ANGRI SANDY LOAM (freeze cycle)

°C	K_a	θ_{uf}
0	23.5	38.1
-0.08	12.9	24.5
-0.30	9.20	18.4
-0.55	8.22	16.6
-0.70	7.47	15.0
-1.03	5.84	11.3
-1.47	5.76	11.1
-2.03	5.60	10.7
-2.55	5.44	10.2

From gravimetric sampling:

ρ_b	1.40
θ at 0°	41.5

GARDEN SANDY LOAM (freeze cycle)

	(1)		(2)		(3)*		
°C	K_a	θ_{uf}	K_a	θ_{uf}	K_a	θ_{uf}	$\bar{\theta}_{uf}$
0	19.4	33.2	29.2	41.1	19.7	33.5	35.9
-0.12	10.5	20.6	13.9	25.9	10.4	20.4	22.3
-0.25	8.0	16.2	8.9	17.8	8.1	16.4	16.8
-0.38	6.93	13.9	7.29	14.6	7.29	14.6	14.4
-0.60	6.41	12.7	6.67	13.3	7.11	14.3	13.4
-1.10	5.92	11.6	6.16	12.1	6.25	12.3	12
-1.92	5.29	9.8	5.68	10.8	5.68	10.8	10.5
-2.91	4.69	8.1	5.24	9.7	5.29	9.8	9.2

From gravimetric sampling:

ρ_b	1.42	1.30	1.51
θ at 0°C	32.1	42.2	33.9

* undisturbed sample

ALLENDALE SILTY CLAY (thaw cycle)

	(1)		(2)	
°C	K_a	θ_{uf}	K_a	θ_{uf}
-0.11	16.0	28.8	13.4	25.2
-0.33	10.7	20.9	10.8	21.1
-0.59	10.2	20.2	9.82	19.6
-0.68	9.82	19.6	9.61	19.2
-0.88	9.20	18.4	9.00	18.0
-1.10	8.80	17.6	8.80	17.6
-1.49	8.03	16.2	8.41	16.9
-2.06	7.47	15.0	7.84	15.8
-2.55	7.29	14.6	7.84	15.8

From gravimetric sampling:

ρ_b	1.18	1.19
θ at 0°C	53.8	53.2
% sat (Vw/Vv)	96.4	95.8

	(3)		(4)		(5)	
°C	K_a	θ_{uf}	K_a	θ_{uf}	K_a	θ_{uf}
-0.07	15.7	28.4	15.7	28.4	15.5	28.1
-0.25	12.1	23.2	13.4	25.2	11.5	22.2
-0.45	11.1	21.6	11.6	22.4	10.0	19.8
-0.63	10.7	20.9	10.9	21.3	9.20	18.4
-0.90	9.61	19.1	10.0	19.8	8.41	17.0
-1.08	9.20	18.4	9.40	18.8	8.03	16.2
-1.55	8.41	16.9	9.00	18.0	7.47	15.0
-2.08	8.22	16.6	8.03	16.2	6.93	14.0
-3.27	7.75	15.6	7.38	14.8	6.59	13.1

From gravimetric sampling:

ρ_b	1.23	1.31	1.22
θ at 0°C	47.3	33.9	38.4
% sat (Vw/Vv)	87.4	66.3	70.5

AGRI SILTY CLAY (thaw cycle)

°C	K_a	θ_{uf}
0	31.9	47.2
-0.08	15.3	27.9
-0.30	10.5	20.6
-0.55	9.61	19.1
-0.70	9.61	19.1
-1.03	9.40	18.8
-1.47	9.00	18.0
-2.03	8.80	17.6
-2.55	8.22	16.6

Fram gravimetric sampling:

ρ_b	1.03
θ at 0°C	50.3

LE2 SILTY CLAY LOAM (thaw cycle)

^{o}C	K_a	θ_{uf}
0	35.0	50.4
-0.08	15.2	27.8
-0.30	11.6	22.4
-0.55	9.92	19.8
-0.70	9.10	18.2
-1.03	8.80	17.7
-1.47	8.03	16.2
-2.03	7.47	15.0
-2.55	7.20	14.4

From gravimetric sampling:

ρ_b	1.20
θ at 0^{o}	48.8

LE7 CLAY (thaw cycle)

^{o}C	K_a	θ_{uf}
0	34.8	50.2
-0.08	14.7	27.0
-0.30	12.6	24.0
-0.55	11.5	22.2
-0.70	11.1	21.6
-1.03	10.4	20.4
-1.47	9.71	19.3
-2.03	9.10	18.2
-2.55	8.60	17.3

From gravimetric sampling:

ρ_b	1.05
θ at 0^{o}	51.3

ELLWOOD CLAY LOAM (thaw cycle)

°C	K_a	θ_{uf}
-0.20	15.07	27.6
-0.35	12.85	24.4
-0.77	10.61	20.8
-1.30	9.45	18.8
-2.12	8.67	17.40
-3.15	7.77	15.6

LE2 and LE7 from suction moisture (Williams 1975)

	LE2		LE7	
°C	w (gg-1, %)	θ^*_{uf}	w (gg-1, %)	θ^*_{uf}
-0.1	22.5	27.0	29.0	30.5
-0.2	18.8	22.5	27.0	28.4
-0.3	17.5	21.0	24.5	25.7
-0.4	16.3	19.5	23.0	24.2
-0.5	15.0	18.0	22.0	23.1
-0.9	13.0	15.6	19.0	20.0

* $\theta_{uf} = w \times \rho_b$ where ρ_b = 1.2 for LE2 and 1.05 for LE7
w is best fit

LOEB CLAY LOAM (thaw cycle)

°C	K_a	θ_{uf}
0	31.2	46.6
-0.25	13.7	25.6
-0.45	12.0	23.1
-0.82	10.4	20.4
-1.25	9.30	18.4
-2.82	8.22	16.5

From gravimetric sampling:

ρ_b	1.19
θ at 0°C	47.5

APPENDIX III

As indicated in section 3.3, a maximum possible ice content for a soil can be inferred if temperature and the freezing characteristic for the soil at normal dry bulk density are known. A nomogram (Figure 19) shows the relationships between ice content, θ_{uf} and ρ_b. The program listed in this Appendix can also be used to determine ice content from a measured θ_{uf} (i.e., Ka); the required input information at a given temperature (T_1) is:

1. dry bulk density for the normal soil;
2. particle density;
3. θ_{uf} at T_1 for the normal soil

The program then calculates the volume of solids, ice and water for the normal soil at T_1. By inputing θ_{uf} at T_1 for a sample with excess ice, the program will calculate the dry bulk density and ice content. It should be emphasized that the ice content determined via the nomogram or the program is the 'maximum possible' value and not an actual measured value.

A program listing (in BASIC) and a sample output follows.

```
M1= MASS OF SOLIDS
M2= MASS OF WATER
B1= BULK DENSITY
V1= TOTAL VOLUME
V2= VOLUME OF SOLIDS
P1= PARTICLE DENSITY
V3= VOLUME OF VOIDS
V9= VOLUME OF H2O MAXIMUM ASSUMING %SAT OF 95%
V8= VOLUME OF AIR

T1= SAMPLE TEMPERATURE
U1= UNFROZEN VOL. H2O CONTENT FOR THE NATURAL SOIL
V4= VOLUME OF H2O BASED ON U1
V5= VOLUME OF ICE BASED ON U1
U2= VOL. ICE CONTENT BASED ON U1
W1= GRAV. UNFROZEN H2O CONTENT BASED ON U1

B2= UNKNOWN BULK DENSITY
U9= MEASURED VOL. H2O CONTENT FOR UNKNOWN SOIL I.E. EXCESS ICE
M9= MASS OF SOLIDS BASED ON B2
M8= MASS OF H2O BASED ON B2
V6= VOLUME OF SOLIDS BASED ON B2
V7= VOLUME OF ICE BASED ON B2
U8= VOL. ICE CONTENT BASED ON B2


THESE ARE THE SOIL CONDITIONS FOR THE
SOIL FROM THE 0UF-TEMP RELATIONSHIP

BULK DENS.=      1.3000        V. TOTAL=        1.0000
PART. DENSITY=   2.7000        V. SOLIDS=       0.4815
M. SOLIDS=       1.3000        V. H20 MAX=      0.4926
V. VOIDS=        0.5185        V. AIR=          0.0259

     THESE ARE THE CONDITIONS FOR 0UF FROM FREEZING CHARACTERISTIC

     TEMP=      -0.1000         0UF=             0.2000
     V. H2O=     0.2000         V. ICE=          0.3191
     VOL ICE CONT=  0.3191      GRAV H2O CONT=   0.1538


     THE FOLLOWING VALUES ARE FOR THIS SAMPLE
     0UF=  0.1000
     BULK DENS.=  0.6500        MASS OF SOLIDS=  0.6500
     MASS OF H20=  0.1000       VOL. OF SOLIDS=  0.2407
     VOL. OF ICE=  0.6333       ICE CONTENT=     0.6333


     THE FOLLOWING VALUES ARE FOR THIS SAMPLE
     0UF=  0.0500
     BULK DENS.=  0.3250        MASS OF SOLIDS=  0.3250
     MASS OF H20=  0.0500       VOL. OF SOLIDS=  0.1204
     VOL. OF ICE=  0.8037       ICE CONTENT=     0.8037
```

```
  PRINT    ULS OF THE VOLUMETRIC UNFROZEN WATER CONTENT, OBTAINED FROM
  PRINT    TDR TECHNIQUE.  THERE ARE SOME ASSUMPTIONS WHICH MUST BE MADE"
  PRINT  CLEAR; THEY ARE:"
  PRINT  1. TOTAL SAMPLE VOLUME IS FIXED"
  PRINT  2. THE % SATURATION (95%) IS FIXED (APPROX FIELD CAPACITY"
  PRINT
  PRINT  THE ICE CONTENTS CALCULATED WILL REPRESENT PROBABLE MAXIMUM VALUES"
  PRINT
  0 PRINT "M1= MASS OF SOLIDS"
  0 PRINT "ML= MASS OF WATER"
  0 PRINT "B1= BULK DENSITY"
  0 PRINT "V1= TOTAL VOLUME"
  0 PRINT "V2= VOLUME OF SOLIDS"
  0 PRINT "P1= PARTICLE DENSITY"
  0 PRINT "V3= VOLUME OF VOIDS"
  0 PRINT "V9= VOLUME OF H2O MAXIMUM ASSUMING %SAT OF 95%"
  0 PRINT "V8= VOLUME OF AIR"
  0 PRINT
  0 PRINT "T1= SAMPLE TEMPERATURE"
  0 PRINT "U1= UNFROZEN VOL. H2O CONTENT FOR THE NATURAL SOIL"
  0 PRINT "V4= VOLUME OF H2O BASED ON U1"
  0 PRINT "V5= VOLUME OF ICE BASED ON U1"
  0 PRINT "U2= VOL. ICE CONTENT BASED ON U1"
  0 PRINT "W1= GRAV. UNFROZEN H2O CONTENT BASED ON U1"
  0 PRINT
  0 PRINT "B2= UNKNOWN BULK DENSITY"
  0 PRINT "U3= MEASURED VOL. H2O CONTENT FOR UNKNOWN SOIL I.E. EXCESS ICE"
  0 PRINT "M9= MASS OF SOLIDS BASED ON B2"
  0 PRINT "M8= MASS OF H2O BASED ON B2"
  0 PRINT "V6= VOLUME OF SOLIDS BASED ON B2"
  0 PRINT "V7= VOLUME OF ICE BASED ON B2"
  0 PRINT "U8= VOL. ICE CONTENT BASED ON B2"
  0 PRINT
  0 FIXED 4
  0 DISP "INPUT B DENS, VT, P DENS"
  0 WAIT 2000
  0 INPUT B1,V1,P1
  0 M1=B1*V1
  0 V2=M1/P1
  0 V3=V1-V2
  0 V9=V3*0.95
  0 V8=V3-V9
  0 PRINT "THESE ARE THE SOIL CONDITIONS FOR THE "
  0 PRINT "SOIL FROM THE 0UF-TEMP RELATIONSHIP"
  0 PRINT
  0 PRINT "BULK DENS.= ";B1;TAB30"V. TOTAL= ";V1
  0 PRINT "PART. DENSITY= ";P1;TAB30"V. SOLIDS= ";V2
  0 PRINT "M. SOLIDS= ";M1;TAB30"V. H20 MAX= ";V9
```

```
   INPUT T1,U1
   V4=U1/.4
   V5=(1--V4)*1.0905
   U2=V5/V1
   W1=V4/M1
   PRINT TAB5"THESE ARE THE CONDITIONS FOR 0UF FROM FREEZING CHARACTERISTI
   PRINT
   PRINT TAB5"TEMP= ",T1,TAB30"0UF= ",U1
   PRINT TAB5"V. H20= ",V4,TAB30"V. ICE= ",V5
   PRINT TAB5"VOL ICE CONT= "U2,TAB30"GRAV H20 CONT= ",W1
   DISP "ENTER 0UF MEASURED"
   WAIT 1500
   INPUT U9
   B2=U9*W1
   M9=B2*V1
   M8=M9*W1
   V6=M9/P1
   V7=V1-(M8+V8+V6)
   U8=V7/V1
   PRINT
   PRINT TAB5"THE FOLLOWING VALUES ARE FOR THIS SAMPLE"
   PRINT TAB5"0UF= "U9
   PRINT TAB5"BULK DENS.= "B2,TAB30"MASS OF SOLIDS= "M9
   PRINT TAB5"MASS OF H20= "M8,TAB30"VOL. OF SOLIDS= "V6
   PRINT TAB5"VOL. OF ICE= "V7,TAB30"ICE CONTENT=    "U8
   DISP "MORE POINTS? YES=1 NO=2"
   WAIT 1500
   INPUT I
   IF I=1 THEN 930
   DISP "SOIL TYPE THE SAME? YES=1 NO=2"
   WAIT 1500
   INPUT J
   IF J=1 THEN 880
   IF J=2 THEN 360
   DISP "IS TEMP THE SAME YES=1 NO=2"
   WAIT 1500
   INPUT K
   IF K=1 THEN 640
   IF K=2 THEN 520
   END
```